Prototyping a Digital Communication System with a Novel Blind Adaptive Equalizer

Jeff Parker
Dept. of Electrical Engineering
Arkansas Tech University
Russellville, AR 72801
Email: jeff.parker@gmail.com

Ping Liu
Dept. of Electrical Engineering
Arkansas Tech University
Russellville, AR 72801
Email: pliu@atu.edu

Abstract—In this paper, an undergraduate research project, sponsored by Arkansas NASA Space Grant, is presented. A novel blind adaptive channel equalizer, which is based on constant modulus algorithm (CMA) with variable step size, is first proposed. The step size is designed to follow an exponential function, with a large initial value for fast convergence rate and a small steady-state value for a low convergence level. A comprehensive digital communication system is further implemented, which includes a digital modulator for the source signal, a multipath channel, a noise generator and a blind equalizer. The prototype builds around software defined radio concepts on both a PC and a DSP platform. It provides flexibility for comparing different equalization algorithms in various realistic environments. Both off-line simulation in Matlab and real-time experiment on DSP board are conducted and presented. It is observed from the simulation results that the proposed blind equalizer performs significantly better than the conventional fixed-step CMA approach.

I. INTRODUCTION

Wireless communications between remote locations often encounter multipath fading channels which introduce intersymbol interference (ISI) in the received signal. ISI is the one of the major impairments to a single-link communication system. To effectively counteract ISI, an equalizer is required at the receiver end. The constant modulus algorithm (CMA) is arguably the most widespread blind channel equalization principle [1],[2], especially when source alphabets from a M-PSK constellation. Since proposed by [1],[2], CMA has been extensively studied. An excellent review about this algorithm can be found in [3]. The convergence property of CMA has been analyzed [4]. Connection between CMA and Wiener receivers are also build based on a novel geometrical concept [5]. It has been proved that the zero cost can be achieved by this criterion under some conditions [3].

As is well known, the constant modulus criterion employs the constant modulus (amplitude) property of modulated signals. It involves fourth order statistics and its cost function is highly nonlinear with respect to the receiver. Therefore, CMA based equalizer is typically implemented in a gradient-based adaptive manner. The adaptive CMA equalizer suffers a few drawbacks. The CMA algorithm is dependent on the initialization setting of the equalizer. The convergence can be severely slowed down, if the equalizer is initialized to be in the vicinity of saddle points [3]. A simply large step size can not solve this problem. Actually, the step size must be carefully selected to ensure a stable operation while balancing convergence rate and final accuracy (misadjustment or excess mean square error). Moreover, in the stochastic gradient based implementation, the expectation operation on the gradient term is generally dropped for simplicity. This rough approximation generally leads to slow convergence and poor misadjustment, even if the step size is carefully selected.

Clearly, the step size, which typically assumes a constant value, is a critical factor affecting the performance of the adaptive CMA equalizer. In this project, we propose to improve the gradient based CMA equalizer by adopting a variable step size. An exponential function is carefully designed to adjust the value of the step size, where a large value is obtained in the initial stage to bring the algorithm faster to the vicinity of its global minimum point, while a small step size is obtained to reduce the excess mean-square error (MSE) after convergence. The efficiency of the proposed approach is first verified in Matlab on PC. It is further collaborated after being implemented on TI TMS320C6713 DSP board in C.

II. SYSTEM MODEL

Consider a widely adopted input/output model in a communication system with a blind equalizer [5], which is shown in Fig. 1:

\[ x(n) = Gs(n) + w(n) \]  

(1)

where \( s(n) \in \mathbb{C}^m \) is the complex source vector from MPSK, QAM, or PAM modulated constellation, \( G \in \mathbb{C}^{p \times m} \) is a Toeplitz matrix constructed from multipath channel vector \( g \), \( w(n) \in \mathbb{C}^p \) represents additive white Gaussian noise (AWGN), \( x(n) \) is the received signal. The equalization is performed by designing an equalizer \( f \in \mathbb{C}^p \) whose output \( y_n \) is expected
to be an accurate estimate of one of the elements in $s$
\[ y_n = f^H x(n) = u^T s(n) + f^H w(n) \quad (2) \]
where $(.)^T, (.)^H$ stands for transpose and Hermitian, $u^T = f^H G$ is the combined response of the channel and the equalizer. Perfect equalization can be achieved in the absence of noise if $u$ has only one non-zero element [6],
\[ u = e^{i\theta}[0, \ldots, 0, 1, 0, \ldots, 0]^T \quad (3) \]
The position of the non-zero element in $u$ stands for the delay and $\theta$ is the phase shift. Therefore the delay and phase ambiguity are inherent in blind equalization. Different criteria can be used to obtain the equalizer. The CMA criterion seeks to minimize the dispersion of the equalizer output about a constant $r$
\[ J(f) \triangleq E\{|y_n|^2 - r|^2\} \quad (4) \]
where "$E$" represents expectation operation. The constant can be chosen as $r = \frac{E|s|^2}{E|n|^2}$ [1].
Due to the high non-linearity of the cost function, CMA algorithm is usually implemented by stochastic gradient descent method
\[ f(k+1) = f(k) - \mu \Delta J(f) \quad (5) \]
where $\mu$ in the update represents step-size parameter, and the gradient is calculated as
\[ \Delta J(f) \triangleq E\{|y_k|^2 - r|^2\} y_k^* x(k) \]. To reduce the computation complexity, the gradient is generally approximated by its instantaneous value, resulting in the following update for the equalizer
\[ f(k+1) = f(k) - \mu(|y_k|^2 - r) y_k^* x(k) \quad (6) \]
where $\ast$ represents conjugate.

### III. Proposed Variable-Step CMA

The step-size parameter $\mu$ is assumed a constant value in the conventional CMA algorithm. However, it is found in [7] that the system performance depends on the choice of $\mu$ greatly. Generally speaking, a larger $\mu$ results in a faster convergence rate, but higher variance or MSE in the steady state after convergence. A smaller $\mu$ yields a slower convergence rate, but lower MSE after convergence. This motivates us to optimize $\mu$ by adopting different values in the convergence process. In particular, a larger step size is set initially to speed the convergence up. The step size is then gradually decreased to a smaller value to obtain smaller MSE after convergence.

The formula we proposed to adjust the step size $\mu$ is given by:
\[ \mu = \alpha \mu_{opt}(\beta e^{-n/\gamma} + 1) \quad (7) \]
where $\mu_{opt} \triangleq \frac{1}{30 E\{|s|^2\}}$ is the step size suggested by [1], $E\{|s|^2\}$ stands for the signal power, $\alpha$, $\beta$, and $\gamma$ are the introduced parameters. According (7), the step size at the steady state (i.e. for large $n$) can be approximated as $\alpha \mu_{opt}$. Since $\mu_{opt}$, which is only dependent on data modulation, can be regarded as a fixed value, the steady-state step size is thus mainly determined by the parameter $\alpha$. Therefore, properly choosing $\alpha$ to be a smaller value will be able to yield a lower MSE level after convergence. On the other hand, the initial step size can be approximated as $\alpha \mu_{opt}(\beta + 1)$, where $e^{-n/\gamma}$ is approximated as 1 for very small $n$. Therefore, a larger value should be carefully chosen for $\beta$ to yield fast convergence rate while not causing ill-convergence. The parameter $\gamma$ then determines the decaying rate of the step size, i.e. how fast should $\mu$ decrease from its initial larger value $\alpha \mu_{opt}(\beta + 1)$ to the smaller value $\alpha \mu_{opt}$. We propose to set $\gamma$ according to the convergence rate of the conventional CMA. For example, if the conventional CMA is found to converge around 500 iterations, a small step size is thus desired after 500 iterations to achieve lower variance (MSE). In this case, $\gamma$ may be set to 500, which will cause the step size to decrease to approximate $1/3$ of its initial value after 500 iterations.

### IV. Implementation and Simulation Results

#### A. Matlab Implementation

The complete baseband systems featuring both the original CMA equalizer and the proposed variable-step CMA equalizer are first simulated and compared in Matlab on PC. The PAM modulation is assumed for the source sequence. Each symbol in the sequence takes value from the signal constellation $\{-0.1, 0.3, 0.5\}$ randomly. For each realization, 4000 symbols are generated randomly. Fig. 2 (a) shows the transmitted sequence in one realization. It is seen that the sequence is well separated to 6 straight lines, each of which corresponding to one of the 6 values in the signal constellation. The symbols are transmitted through a multipath fading channel with 4 paths. The channel parameters, which are taken from [6], are given by $[-0.4 0.84 0.336 0.134]^T$. Due to the ISI introduced by the multiple paths of the channel, the received signal is highly distorted, which is further corrupted by the noise from the devices. The noise is typically modeled as additive white Gaussian noise (AWGN) in the communication system. Consequently, the received symbols are all mixed up, as shown in Fig. 2 (b), and are impossible to separate without the aid of an equalizer.

An adaptive equalizer is employed to the received symbols in order to suppress the ISI. The equalizer is assumed the structure of an transversal FIR filter with a length of 12 taps. Except that the mid-tap is initialized to 1, all the other taps are initialized to 0. The conventional CMA equalizer is then updated according to eq. (6), with the step size fixed. Various step sizes are assumed for the conventional CMA in the experiments. The proposed variable-step CMA equalizer, denoted as VS-CMA, is also updated according to eq. (6),
where the step size $\mu$ is replaced by a variable one in eq. (7). $\alpha$ is set to 0.05 for a smaller MSE after convergence. $\beta$ is set to 20 such that the initial step size, which is approximate $\mu_{opt}$, is still large enough for a fast convergence. The parameter $\gamma$ determining the decaying rate is set to 500. Fig. 3 shows the step size v.s. iteration number for the proposed approach and the conventional CMA.

We first consider a communication link with 25dB signal to noise ratio (SNR). The SNR is defined as

$$\text{SNR in dB} = 10 \times \log_{10} \frac{\text{transmitted signal power}}{\text{noise power}}.$$ 

The intersymbol interference is first used as performance indicator, which is defined as

$$\text{ISI} = \frac{\sum_{l} |u_l|^2 - |u_{l_{max}}|^2}{|u_{l_{max}}|^2}$$

where $u^T = \text{conv}(f,g)$ denotes the combined response of the channel and the equalizer, $|u_{l_{max}}|$ is the maximal absolute value of all elements in $u$. Clearly, when $u$ has only one nonzero component as in (3), ISI=0, which is the the ideal situation. Small ISI indicates the proximity to the desired situation.

The simulation results based on 100 Monte Carlo runs are plotted in Fig. 4. It is observed that the proposed variable-step CMA (VS-CMA) shows the fastest convergence rate and the lowest ISI level, indicating smaller residual error. In contrast, the conventional CMA, has either a fast convergence but a very high ISI level (such as $\mu = 0.14$), or a lower ISI level but much slower convergence rate. Fig. 5 further plots the output of different equalizers, where the first three subplots correspond to the CMA equalizer with $\mu = 0.14$, $\mu = 0.05$, and $\mu = 0.014$, respectively, and the last subplot is for VS-CMA. It is observed that the output of each equalizer is separated into 6 beams, corresponding to the 6 values of the input sequence. The output of the proposed VS-CMA in Fig. 5 (d), is found to be separated very well from about the 300th iteration, which is much faster than the conventional CMA with $\mu = 0.05$ and $\mu = 0.014$. On the other hand, the output beams of the proposed VS-CMA also show the least variance, which is much smaller than that of the CMA equalizer with $\mu = 0.14$. Fig. 4 and Fig. 5 verify that the proposed approach outperforms the conventional CMA significantly.

Since the probability of detection error is especially meaningful in the communications context, we then use it as a performance indicator for comparing different equalizers. We consider a more complex situation with variable input SNR from 0dB to 35dB. The probability of detection error is obtained from multiple independent realizations with random input signals and defined as the percentage of accumulated detection errors among total number of transmitted symbols up to the current iteration. As in the previous simulation, 100 Monte Carlo runs are executed for obtaining the average detection errors for different equalizers. The results are plotted in Fig. 6. It is seen that for all examined SNRs, the proposed VS-CMA demonstrates the best performance by yielding the lowest probability of detection error, especially in the high SNR region.

**B. DSP Implementation**

The equalizer in a practical application may be implemented in either ASIC or DSP, where DSP implementation has received more and more attention in recent years. According to [8], more and more components, which were previously implemented in ASIC in the 2G or 2.5G handsets, have been or will be shifted to programmable DSPs in the 3G and newer generations of wireless communication products. We thus consider the DSP implementations of both the proposed CMA and the conventional CMA algorithms and compare them in the more practical DSP platform. We adopt the TI TMS320C6713 DSK, which is a float-point development board.

A set of C programs are developed on the DSP board to simulate the baseband system including the transmitter, multipath channel, and the CMA equalizer. The channel parameters are assumed the same as before. For simplicity and ease of illustration, we adopt BPSK source for the transmitted sequence, i.e. the signal constellation is $\{1, -1\}$. Ideally, if the equalizer can remove ISI completely, then its output $y_n$ in the absence of noise, should take the value of 1 or $-1$. Correspondingly, $y_n^2$ should have the constant value of 1. However, any nonperfect equalizer and the presence of noise will cause $y_n^2$ deviated from the constant value of 1. The smaller deviation indicates the better performance of the equalizer. Therefore, we use a new criterion, which is the dispersion of $y_n^2$ from 1, to evaluate the performance of the equalizer for DSP implementation. The dispersion here is denoted as MSE and defined as below.

$$\text{MSE}(n) = (y_n^2 - 1)^2.$$

MSE is much easier to evaluate on the DSP board than the performance indicators used before. Clearly, smaller MSE indicates better performance and is more desired.

While implementing the algorithms on the DSP board, we adopt both a large and a small step size for the conventional CMA equalizer and an exponential step size for the proposed VS-CMA. The step sizes are plotted in Fig.7. The exponential step size for the proposed approach is generated on PC first and then exported to the DSP program as a “.cof” file. Fig. 8 shows the MSE results. It is seen that the CMA equalizer with large step size converges very fast but with large MSE level. The CMA equalizer with small step size then converges slowly. In contrast, the proposed VS-CMA shows both fastest
convergence rate and lowest MSE level. DSP implementation thus verifies the effectiveness of the proposed VS-CMA.

V. CONCLUSIONS

In this paper, an improved constant modulus based equalizer is proposed in the presence of multipath. The method adopts variable step size to achieve fast convergence rate as well as low convergence level after convergence. The proposed method is simulated in Matlab and also implemented on the real-time DSP board. Experiment results verify the superiority of the proposed variable-step CMA to the conventional CMA.

REFERENCES

Fig. 5. Output of different equalizers at 20dB noise.

Fig. 6. BER comparison.

Fig. 7. Step size adopted for different approaches for DSP implementation.

Fig. 8. MSE comparison based on DSP implementation.