Mathematics Learning Outcomes for Engineers in an Age of Excel®, MATLAB®, etc: Some Observations and Thoughts

By

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Abstract

Having spent a career in industry, about a decade ago, I looked forward to the prospect of teaching courses on Design of Experiments, Statistical Process Control, and statistics. I was then and still am impressed with the enthusiasm, aptitude and accomplishments of the students I teach, whether they be at a university or at a technical conference.

Overall, I believe that the students of today are much better prepared for the world of engineering than I was at their age. However, in teaching them over these past few years, I have uncovered some weaknesses in their math skills, that I had initially missed. These weaknesses, I believe are the result of the availability and use of power math tools, such as Excel®, MATLAB®, MINITAB®, etc. This paper is not to suggest that these tools are not useful, on the contrary, they form a foundation for modern engineering and science. However, their availability and power may be some of the cause of the math weaknesses I observed. This paper also does not suggest that these students are less able or hardworking than in the past.

As a result of these observations, I am now teaching my senior/graduate level statistics courses by adding some assignments in which the students perform calculations by hand, and yes even with slide rules. This paper will review what I have learned from this experience and some thoughts on how to address this engineering education need for today’s students.

Introduction: My First Experience

It was in the early 1990s and I was responsible for the optical interface in IBM’s ECSL (Early Corporate Serial Link) optical networking technology. In this assignment I had to assure that a certain amount of light would be “coupled” from the transmitting laser into the receiving fiber, too little light and the signal would be too weak to travel 20 kilometers, too much light and it would violate laser safety requirements. Unfortunately, due to the small size of the fiber core,
about 9 micrometers, geometric optics does not suffice. Therefore, the modeling of this phenomenon requires numerical integration of laser optical modes to the fiber optical modes. Figure 1, is a physical schematic of this system.

![Figure 1. The coupling of light into an optical fiber](image)

Fortunately, I had discovered a young PhD at an optical research laboratory who had developed a computer program to calculate the coupling of light in such optical systems. My product specifications were such at I could tolerate a 3 dB power loss within the mechanical constraints of the system. Hence, I needed to know to what tolerances I needed to hold L, F, O1, O2 and the mode filed diameter of the laser and fiber. As the type of mechanical tolerances required would be in the single micrometer range, there was a question as to whether or not this optoelectronic system could be developed at an acceptable cost.

So I visited the optical research lab with great hopes. After introductions, I asked the researcher, who had developed the sophisticated software, questions relating to how the coupling of the light is affected by varying L, F, O1, O2 and the mode filed diameter of the laser and fiber. He had no idea. He told me if I gave him specific values for these parameters, he could tell me what the percent of light coupled would be. I was stunned that he had not used his software program to develop graphs so that he could discuss, in general, the effect of these parameters on the coupling of light. E.g. “The coupling of light drops off as the square of O1 and O2, with a 50% loss of power typically at 4 micrometers or so.” Needless to say, I was surprised at his lack of curiosity in these relationships or perhaps his lack of understanding that knowing, approximately, these functional relationships would be profoundly valuable to his “customers” and secure the importance of his software program. It wasn’t enough just to have the computer program available, one needed to use this digital tool to develop analog graphs to aid designers. This experience was the first I recall in which I observed a person being an intellectual slave to his technology. This servitude to the software was a portend of things to come.
Math Skills Examples: Ten Years Later

By the early 2000s I was regularly teaching, designed experiments (DOE), statistical process control (SPC) and materials science at several universities and at conferences. Most of the classes that I teach have a majority of graduate students. These graduate students obtained their Bachelor’s degrees in engineering from educational institutions in the United States and abroad. So, I believe the observations I will be discussing are not peculiar to an isolated group of students, but are a world-wide phenomenon.

In teaching statistics, I use Montgomery and Runger’s *Statistics and Probability for Engineers*, 4th edition. I find the book to be a good mix between the practical and theoretical. The book requires only an occasional use of calculus and as I tell my students, “Statistics is the most useful mathematics that is not too difficult.” So, when I first taught this course, my expectation was that any calculus or differential equations that would be covered would be easy for the students. My first surprise came on the mid-term exam. On this exam, I asked that the students to sketch an approximate graph of a continuous probability density function, $Ce^{-x}$, and solve for the constant C, as seen below.

$$\int_{0}^{\infty} C e^{-x} \, dx = 1$$

A significant minority, about 15-20%, of the students could not solve for C, and about 5-10% could not plot the general shape of the $Ce^{-x}$. I found this fact surprising, for students at a senior or graduate level. At this point, I decided to add a little calculus refresher to my lectures. I felt this was especially important as this course was the last mathematics course for considerably more than half of the students.

The next surprise came when I assigned the homework problem: using calculus, derive that the volume of a cone is $\frac{1}{3}\pi r^2 h$. Not only did the students struggle with this problem, but my four teaching assistants came to me and said that they did not know how to solve it.

A year later, in the same course, I assigned a problem to analyze Eddington’s quote which follows:

*If I let my fingers wander idly over the keys of a typewriter it might happen that my screed made an intelligible sentence. If an army of monkeys were strumming on typewriters they might write all the books in the British Museum.*

I wanted the students to calculate the likelihood of such an army of monkeys typing, by chance, *Gone with the Wind* (GWTW). I suggested, that the students consider a keyboard to have 100 letters, including upper case, numbers etc. In their solution, they should then fill our 13.7 billion
light-year radius universe with monkeys and let the monkeys type, at one character per second, for the age of the universe. The students could assume that GWTW has 1 million characters. Since each correct key has a likelihood of $10^{-2}$ of being struck, the probability of one million keys being correctly struck by one monkey typing GWTW in one attempt is $10^{-2,000,000}$.

Assumptions will make the number of copies that all of the monkeys can type in the age of the universe vary, but the order of magnitude is $10^{100}$ copies. So the chance of all of these monkeys typing one copy of GWTW in the age of the universe is about $10^{-2,000,000} \times 10^{100} = 10^{1,999,900}$.

While attempting to solve the problem, several students sent me emails saying something like, “$10^{-2,000,000}$ is a very small number,” to which I agreed. It didn’t occur to me, until I received their homeworks, that because Excel® could not handle $10^{-2,000,000}$, the students assumed the problem couldn’t be solved.

More recently, while developing a statistics exam, I needed a final ten point problem. I thought I might give the students and easy last problem, so I gave them one, similar to problem 10 in the appendix. This problem asks, “If 1000 males are alive at 20 years of age and their death rate is defined by the graph, estimate how many will be alive at 70 years of age.” The problem requires either reading the death rate for each year and applying it, a rather laborious task, or approximating the average over a decade, say in each decade’s 5th year and applying it only 5 times. Only about 10% of the students choose a reasonable approach. Most of them simply took the death rate at age 70 (about 2%) and applied it one time. This approach gave them an erroneous answer of 980 of the 1000 surviving to 70. The unreasonableness of this answer, from experience, did not seem to register with the students who missed the problem.

Shortly after this experience, I had an opportunity to examine about six of the stronger students orally in math. I told the students beforehand that I would cover “the headlines” of freshman calculus in my examination, in addition to statistics.

There were two problems that I posed, that all of the students performed poorly on. The first was: $\lim_{x \to 0} \frac{\sin x}{x} = 1$.

It surprised me, on several levels, that the students struggled with this problem. One surprise was that most of the students did not remember to use L’Hospital’s Rule. The other, and more astonishing thing to me was that none of the students knew that $\sin x \approx x$ for small $x$. In discussing this problem with them during the examination, none of them also seemed to know of the power series expansion for $\sin x$.

The next problem was: What is $e^{-500} \pi^{430}$ in scientific notation?
The students were allowed a calculator, but these small and large numbers a calculator cannot handle. Most students could not do the problem, even though they recognized that it was high school math.

A reasonable approach would be to find the base ten logs of e and π, multiply by -500 and 430 respectively, add the results and then take the antilog to determine that the answer is about: 4.24x10^{-4}.

Thoughts on Why Some Students have These Math Deficiencies

All of this anecdotal evidence begs the question of why a troubling number of engineering seniors/graduate students appear to have major weaknesses in their math skills. I will now pose some thoughts on this concern.

Malcolm Gladwell proposed in his book *Outliers*, that to become an expert in something, one needed to invest 10,000 hours in “deliberate practice.” My thoughts are that this proposition is at the crux of the issue. With modern software tools, a student simply does not need to spend the time working on math, it is done for him. Even symbolic integration, can be accomplished with many software tools, such as MATLAB®, MAPLE®, etc.

In addition, I think there is a reluctance to have students perform drill. As an example, a requirement for sophomore dynamics class, when I was a student, was to convert the Lagrangian (∇^2) from Cartesian to cylindrical and spherical coordinates. I just couldn’t do it without practicing it over and over again. I doing so, even at the time, I could see the benefit, as the process clearly added to my math skills.

Consider also the following example. Let’s say a student has to plot e^{-x}sin x from 0 to 4π. He will likely go to Excel®, enter into one column data points from 0 to 4π and then enter the Excel® formula for e^{-x}sin x in the next column and then have Excel® plot it. In less than a minute he will have the graph. When I was a student, decades ago, this same task would have taken 20 minutes or so. I would likely have made a table by hand, replicating the columns that Excel® made for today’s student. The calculations would have been performed with a slide rule. I would have then plotted the graph on graph paper. All of these operations gave me a closer connection to the equation and resulting graph, such that it stayed in my memory. Even these many years later, the shape of the curve is with me, for my younger colleague of today, the shape of the curve was never with him. Situations like this in today’s digital environment, make it more of a challenge for our students to have strong and intuitive math modeling skills.

Another benefit of using the slide rule is that approximations become more evident. This may be why so few students seem to be familiar with the fact that sin x ≅ x for small x. If one uses a slide rule to calculate sin x, one quickly learns this truth as it is clear in using the scales.
Almost all of my data are anecdotal. However, I plan to use the problems in the appendix as a
test with volunteers to obtain some statistics. I hope to have the data by mid 2010.

What is to be done?

My thoughts, at this point, are that the following actions would be helpful to assure students have
stronger math skills upon graduation with a BS in engineering:

1. Students should have more paper and pencil drill in math. They should be solving
   problems like those in the appendix and harder, all though their academic career.
2. Students should have some experience graphing on graph paper, especially log graph
   paper. They should also have exercises that require reading complex graphs.
3. Re-introduce the slide rule, strictly for learning. The slide rule is a valuable tool, as there
   is a visual connection to logarithmic calculations.

Conclusion

Powerful mathematical software tools such as Excel®, MATLAB®, MINITAB®, etc will
continue to make a fundamental contribution to engineering and engineering education.
Mastering these types of tools is a requirement for the modern engineer. However, development
of strong math skills benefits from paper and pencil calculating and drill. It is up to we educators
to determine the best mix of these two fundamental approaches to solving problems to best
prepare our students for productive careers as engineers.

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Appendix: Math Skill Expectations at a BS in Engineering Level: A Proposal

1. \( \lim_{x \to 0} \frac{\sin^2 x}{x^2} = \)

2. The following function of \( x \) is to be used in numerous mathematical manipulations for \( x < 0.001 \). Accuracy of 1% is desired. Propose a simpler function of \( x \) to use \( \frac{C}{1 - x} = \)

3. Using calculus, derive that the volume of a cone is \( \frac{1}{3} \pi r^2 h \).

4. \( \int_0^\infty C e^{-2x} \, dx = 1 \), solve for \( C \).

5. Sketch \( e^{-x} \cos x \) for \( x = 0 \) to \( 4\pi \). By inspection, is the integral of this term from 0 to \( \infty \) positive or negative?

6. What is the derivative of \( e^{-x^2} \sin^2 x \)?

7. The base 10 log of e is 0.4343, while the base 10 log of \( \pi \) is 0.4971. What is the base ten log of e times \( \pi \).

8. Integrate \( \int \frac{2x \, dx}{\sqrt{5 - 4x^2}} \).

9. Without using a calculator, estimate the value of this term: \( \frac{200 \sqrt{10}}{\pi^2 \sqrt{30}} \).
This graph plots the death rate per 100,000 per year versus age for males. Assume there is a group of 1,000 males who are 20 years old. Assume also that this graph does not change with time. Estimate how many will survive to be 70 years old.

