AC 2012-2947: APPLYING DYNAMICS TO THE BOUNCING OF GAME BALLS: EXPERIMENTAL INVESTIGATION OF THE RELATIONSHIP BETWEEN THE DURATION OF A LINEAR IMPULSE DURING AN IMPACT AND THE ENERGY DISSIPATED.

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Applying Dynamics to the bouncing of game balls: experimental investigation of the relationship between the duration of a linear impulse and the energy dissipated during impact.

Abstract

This paper discusses experiments done as a class assignment in a Dynamics course in order to investigate the relation between the duration of a linear impulse and the energy dissipated during impact. After analysis had been presented in lecture on the relation between work and energy and on the connection between linear impulse and linear momentum, a series of distinct but related projects was assigned as hands-on applications of the results of analysis.

In project one, it was shown that the height to which a dropped ball rebounded depended upon the height from which it was dropped. The ratio consisting of the rebound height divided by the drop height was found to decrease with increasing drop heights. This pattern held true with basketballs, tennis balls, ping pong balls, volleyballs, and racket balls. In project two, the rebound height of a basketball was investigated as a function of the inflation pressure of the basketball. It was determined that the rebound height increased with increases in the inflation pressures. In project 3, experiments that would allow for the collection of data to help explain the results of projects one and two were designed and carried out.

The relationship between the mechanical energy dissipated by a ball bouncing off a rigid surface and the duration of the impact was investigated analytically and experimentally. Three different kinds of balls were used: basketballs, tennis balls, and ping pong balls. Data were collected using digital cameras and processed using software freely available on the web.

For each of the tested balls, analysis and experimental data agreed. They showed that when the duration of impact increased, so did the amount of energy that was dissipated. Similarly, when the duration of impact decreased, so did the amount of energy that was dissipated. Consequently, for each tested ball, the longer the duration of the impulse, the more energy was dissipated.

1. Introduction

Dynamics is a very challenging course [1-14]. One way to assist students to learn Dynamics is to introduce projects that allow them to apply concepts and results to everyday circumstances [15]. This paper discusses experiments done as a class assignment in a Dynamics course in order to investigate the relation between the duration of a linear impulse and the energy dissipated during impact. After analysis had been presented in lecture on the relation between work and energy and on the connection between linear impulse and linear momentum, a series of distinct but related projects was assigned as hands-on applications of the results of analysis.

In project one, the idea was to gain hands-on experience with the dissipation of energy by using the concept of the coefficient of restitution. Students collected data and showed that the height to which a dropped ball rebounded depended upon the height from which it was dropped. The ratio consisting of the rebound height divided by the drop height was related to the coefficient of restitution and found to decrease with increasing drop heights. This pattern held true with basketballs, tennis balls, ping pong balls, volleyballs, and racket balls [15].

In project two, the rebound height of a basketball was investigated as a function of the inflation pressure of the basketball. It was desired to know how inflation pressure affected the amount of energy that was dissipated during impact. It was determined that the rebound height increased with increases in the inflation pressures. Therefore, increasing the inflation pressure increased the coefficient of restitution. This showed that the coefficient of restitution was a dynamic quantity and explained why the National Basketball Association (NBA) specifies the inflation pressure of balls that are used in games [24].

It has been established experimentally that, for a given drop height, the rebound height depends upon the nature of the ball; and that, for a given ball, the rebound height depends upon both the drop height and the inflation pressure. Therefore, the results of these experiments show that specifying the height from which a basketball is dropped during a ball-drop test and its internal pressure during the subsequent fall is essential in order to interpret the quality of the bounce of that basketball properly and without ambiguity. This information is essential because it is possible to achieve the same rebound height with a given ball by using various combinations of the internal pressure and the drop height [24].

In project 3, one wanted to understand how energy was dissipated when an elastic ball strikes a hard floor and bounces off. To do so, one needs to use concepts that were learned in Dynamics to model the interaction between the ball and the floor during impact.

The impact between the ball and the floor is not perfectly elastic, because it causes a loss of energy. The energy that is dissipated during that process does so through various forms. Examples are heat, sound, vibrations, and the inelastic deformation of the ball itself. Students conducted experiments that produced data to help relate the dissipation of energy during an impact and the linear impulse that is applied to the ball during that same impact. The relationship between the mechanical energy dissipated by a ball bouncing off a rigid surface and the duration of the impact was investigated analytically and experimentally. Three different kinds of balls were used again: basketballs, tennis balls, and ping pong balls. Data were collected using digital cameras and processed using software freely available on the web.

The remainder of the paper is organized in the following manner: first energy dissipated during an impact and the corresponding applied linear impulse are determined analytically; then, the duration of impact, energy dissipation, and the deformation of a ball are related analytically; next, the experimental determination of the duration of impact is discussed and experimental results are presented; after that, conclusions are presented; and, lastly, the impact that the projects had on learning Dynamics is summarized.

2. Energy dissipated and applied linear impulse

Consider a particle that was released from rest from an initial height h_i above the plane of impact. Let that particle strike the plane of impact and bounce vertically upwards to a final height h_f, where it has been determined experimentally that $h_f < h_i$. If air resistance is neglected, then, the relation between the work done on the particle during the free-fall-and-rebound cycle and the change in its kinetic energy over that same cycle requires that negative work be done on the particle during impact[10-15]. The magnitude of that work is given by

$$U = mg\left(h_i - h_f\right). \qquad (1)$$

Experimental data indicate that when the drop height, h_i, is increased, so is the rebound height, h_f. Furthermore, data also show that the drop height increases faster that the rebound height, which increases the difference between the two [6,15]. From Eq. (1), this, in turn, increases the energy dissipated by the impact. It follows from analysis, therefore, that increasing the drop height increases the energy that is dissipated during impact [15,24].

Similarly, from the relation between linear impulse and linear momentum, the vertical impulse that acts on the particle during impact is given by [1-6]

$$\int_{t_b}^{t_a} \vec{F}(t)dt = m\left(\overrightarrow{V_a} - \overrightarrow{V_b}\right), \qquad (2)$$

where the subscript "a" stands for 'after impact' and "b" stands for 'before impact'. Hence, t_a is the time immediately after impact and t_b is that immediately before impact. Similarly, V_a is the speed immediately after impact and V_b is that immediately before impact. Since energy is conserved during the free fall of the ball before the impact occurs and is conserved again during rebound of the ball after impact, the speeds V_a and V_b can be expressed in terms of rebound and drop heights, respectively. Therefore, in scalar form, Eq. (2) can be written as [19-21]

$$\bar{F}\Delta t = m\sqrt{2g}\left(\sqrt{h_i} + \sqrt{h_f}\right), \qquad (3)$$

where $\bar{F}$ is the average magnitude of the impulsive force and Δt is the duration of the impact. It is defined by $\Delta t \equiv t_a - t_b$. The positive sign in Eq. (3) is due to the opposite directions of the velocities of the ball before and after impact.

Experimental data indicate that, when the drop height, h_i, is increased, so is the rebound height, h_f. It follows that, when the drop height is increased, the linear impulse received by the particle during impact also increases [4, 6, 11, 15]. So far, then, analysis indicates that increasing the drop height increases the following quantities: 1) the rebound height, 2) the difference between the drop height and the rebound height, 3) the energy dissipated by the impact, and 4) the impulse applied to the ball during impact by the plane of impact. However, it is not yet clear what happens to either the average impulsive force, $\bar{F}$, that is applied to the particle during impact, or to the duration, $\Delta t,$ of the application of the linear impulse, as one increases the drop height. Clearly, Eq. (3) indicates that one, or the other, or both, of them should increase but it is not clear which is the case [11].

3. Duration of impact, dissipation of energy, and deformation of a ball

It is possible to use what students had learned in the course to find the duration of impact, the deformation of the ball during impact, and the relation between the two. One can use the conservation of energy to relate the duration of the impact to the deformation of a basketball.

One way to do so is to take into account the mass, m, and the elastic stiffness, k, of the ball. For purposes of simplification, one considers the impact of the ball onto the floor of the basketball court as being similar to that of a rigid mass falling from rest from some height onto an unstretched spring that is linear, massless, and resting on a rigid surface.

Accordingly, consider a rigid ball of mass m that rests on a vertical spring of stiffness k. The static deflection, δ_0, of the spring that would be due to the weight of the ball is given by [1]

$$\delta_0 = \frac{mg}{k}, \qquad (4)$$

where g is the local acceleration of gravity. In the absence of damping, the natural period of free vibrations, $\tau,$ of such a mass m that is supported by a spring of stiffness k, is given by [10]

$$\tau = 2\pi\sqrt{\frac{m}{k}}. \qquad (5)$$

This natural period of vibration of the mass, τ, represents an approximation for the duration of the impact process. Eq. (5) indicates that increasing the mass of the ball would increase the duration of impact, for a ball of fixed stiffness. Similarly, increasing the stiffness of the ball would decrease the duration of impact, for a ball of fixed mass. For the case of a basketball, it has been shown that increasing the inflation pressure increases the stiffness of the ball [24]. It follows, from Eq. (5), that this action reduces the duration of the basketball's impact with the court. This result is supported by experimental data [15,24].

Using Eq. (5) as a basis, it was hypothesized that inflation pressures affected the durations of the impacts between a basketball and the floor. To test this hypothesis, students tested the same basketball multiple times; the basketball was progressively inflated to different levels of pressure. Before each test, the ball was inflated to a different level of internal pressure and dropped from the same height. It was found, by direct measurements, that increasing the inflation pressure of a basketball did the following: 1) it reduced the duration of its impact with the floor; and 2) it increased the height to which the ball rebounded. These two results led to the conclusions that increasing the inflation pressure of a basketball must reduce the energy dissipated during the impact and that the duration of impact must be directly related to the dissipation of energy. That is: the longer the duration of impact, the larger the dissipation of energy; and the smaller the duration of impact, the lesser the dissipation of energy [15,20,24]. Eq. (5) can be combined with Eq. (4) to express the static deflection, δ_0, in terms the natural period. Doing so leads to

$$\delta_0 = g\left(\frac{\tau}{2\pi}\right)^2. \tag{6}$$

Let a rigid ball of mass m be dropped from a height h_o above the top of an uncompressed spring of stiffness k, on which it once rested and let the ball subsequently land on that spring; then, δ, the maximum dynamic deflection of that spring, when it is struck by the falling mass, can be determined using the conservation of mechanical energy. It is given by

$$\delta = \delta_0\left(1 + \sqrt{1 + 2\frac{h_0}{\delta_0}}\right). \tag{7}$$

Recognizing that $2h_0/g$ is related to the time it takes the mass to fall freely from rest through a distance of ho, one sets

$$t_f = \sqrt{\frac{2h_0}{g}}. \tag{8}$$

Combining Eq. (7) and Eq. (8), the dynamic deflection, δ, of the spring can then be written in terns of t_f as

$$\delta = g\left(\frac{\tau}{2\pi}\right)^2\left(1 + \sqrt{1 + \left(\frac{2\pi t_f}{\tau}\right)^2}\right). \tag{9}$$

Or, if one uses the drop height, h_0, as a reference, then, Eq. (9) can be rewritten to become

$$\frac{\delta}{h_0} = 2\left(\frac{\tau}{2\pi t_f}\right)^2\left(1 + \sqrt{1 + \left(\frac{2\pi t_f}{\tau}\right)^2}\right). \tag{10}$$

It can be seen from Eq. (10) that τ, the duration of the impact, is related to the deformation of the basketball, δ, in such a way that the larger the duration of impact, the larger the deformation. Since, t_f, the duration of free fall, is generally larger than the duration of impact, it is reasonable to expect that the ratio τ/t_f will be less than "one" in practice, and indeed, very small. Hence, one requires $0 < \tau/t_f < 1$. Eq. (10), which has been illustrated graphically in Figure 1, allows one to infer larger deformations from larger durations of impact, and vice versa [10,15,20,24].

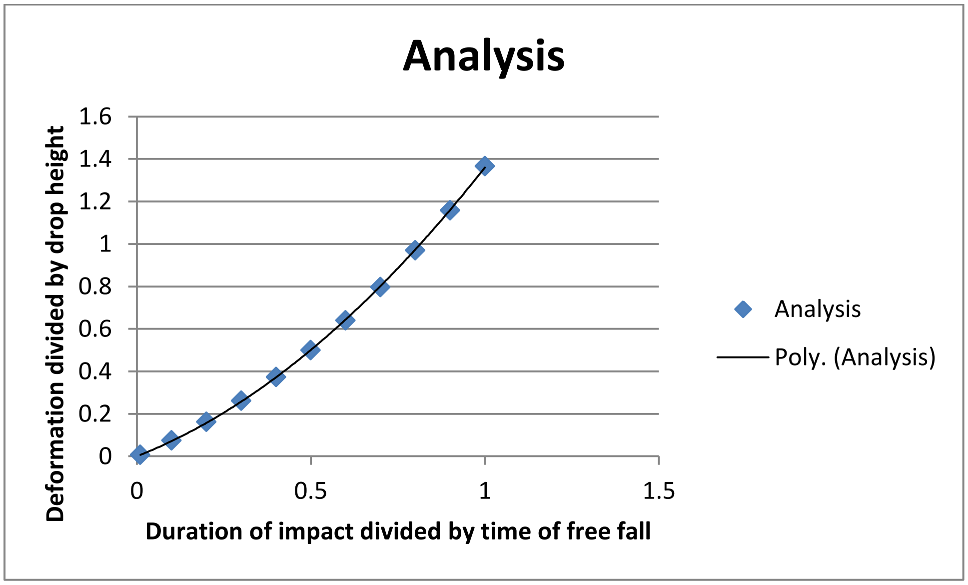

Figure 1. Normalized deformation of the ball during impact vs. normalized duration of impact

4. Experimental determination of the duration of impact

Designing an experiment to measure the duration of impact was a little more complicated than that to determine the rebound height. At first, students tried to use film footage obtained from the measurements of bounce heights. It proved inadequate to measure the durations of impact. They inferred from this failure that the longest duration of impact was shorter than what the camera in use could record. Since the speed of that digital camera was thirty frames per second, they concluded that the duration of the longest impact in their tests was less than 1/30 of a second, or 0.034 seconds [15, 20, 24]. Thus, although unsuccessful in helping students achieve their intended purpose, nevertheless, analyzing film footage helped establish two things: 1) that the durations of impacts that were sought were very small, indeed; and 2) that 0.034 seconds was an upper bound for the magnitudes of those durations.

Students had to change the measurement technique and it was decided to use a microphone and sound recording software to record the duration of the impacts of all three balls. Two types of software were popular among students: Audacity[22] and Goldwave [23]. Hundreds of waves were identified and processed. Students could identify the specific portion of the waveform that corresponded to a given impact; and, by highlighting it, the software displayed the duration of that part of the waveform. It was determined that an impact that lasted 0.0015 seconds could be detected using either software [15, 20, 24].

First, the durations of impact from varying drop heights were tested. The computer's microphone was placed close to where the balls would make contact with the floor and the position of the microphone was at the same spot through all trials. The sound made by the impact of the basketball on the floor was recorded from each of the heights that were used in the experiment. The tests were done in the same manner; first using a basketball, then, a tennis ball, and finally, a ping pong ball [15, 24].

Some practice was needed in using software to view and analyze the recorded sounds. However, it was possible, after some practice, to identify the portion of the waveform that corresponded to an impact relative to background noise because impact caused the intensity of the recorded sound to increase suddenly for a short time. By highlighting the segment of the waveform that corresponded to the impact, the software displayed the start and end times of the wave pulses created by the impact. The duration of the corresponding sound could be calculated by taking the difference between the two displayed times. The results collected from this process are shown in Figure 2. Data in that Figure show bands wherein the durations of impact are constant. This is due to the fact that the corresponding durations were very close to each other but the resolution of the software did not allow for refined separation of the durations of impact from drop heights that were very close to each other. Nevertheless, the general trends are clear: duration of impacts increase as one increases drop heights. For the tested balls, experimental data showed that, when the duration of impacts increased, so did the amount of energy that was dissipated. Similarly, when the duration of impact decreased, so did the amount of energy that was dissipated. It was postulated that when a ball was dropped from a high elevation, its impact with the floor was associated with larger deformations, hence larger times of impact, than when it was dropped from a lower height [24].

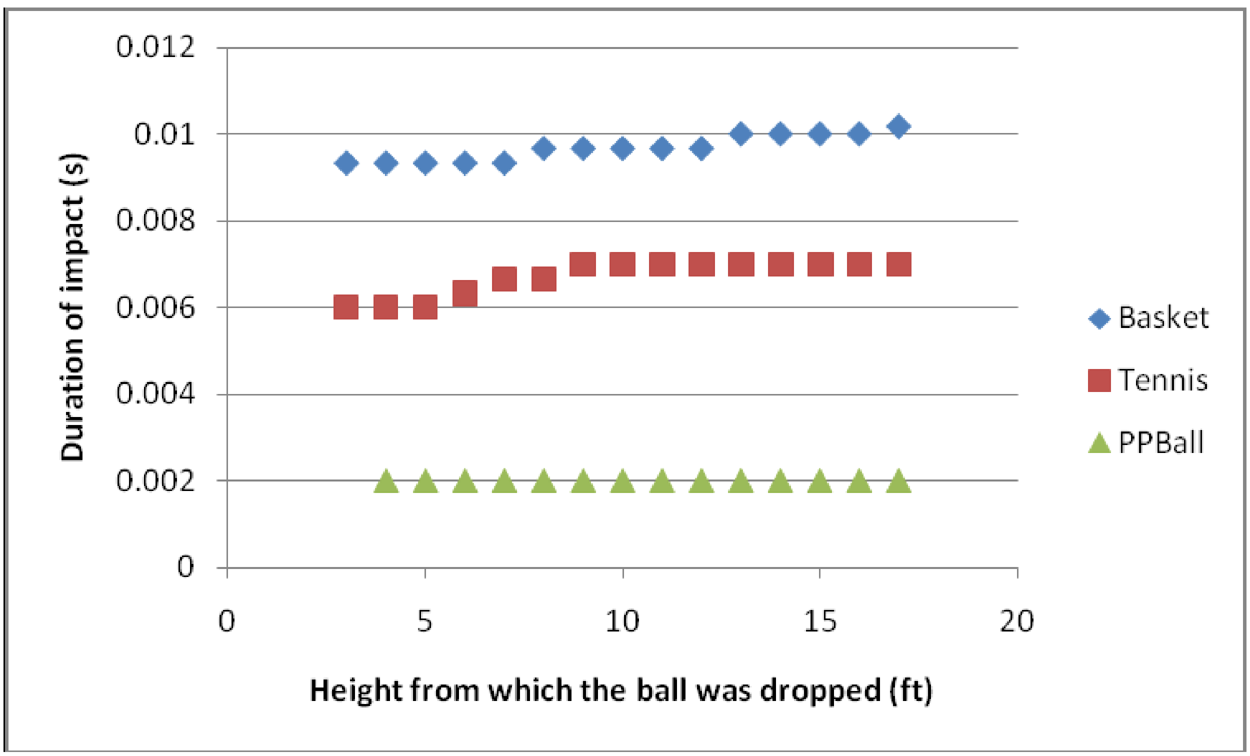

Figure 2: Durations of the impacts vs. drop heights [24].

5. Discussion

Analysis showed that it was reasonable to expect that the deformation of the ball would increase with increases in the durations of impact, Eq. (10). The durations of impact are very small, as one would have expected. According to Eq. (5), stiffer balls were expected to yield smaller durations of impact. That is exactly what experimental data showed: for basketballs, the durations of impact were around 10^{-2} seconds; for tennis balls, durations were around 6.5×10^{-3} seconds; and for ping pong balls, they were around 2×10^{-3} seconds.

It can be seen from Figure 2 that increasing the heights from which the balls were dropped increased the duration of impact noticeably for basketballs. Such increases were moderate for tennis balls and were very hard to detect in the case of ping pong balls. The differences in these behaviors were attributable to the masses and stiffnesses of these balls and to the changes in the kinetic energies that were introduced by increasing the heights from which the balls were dropped, Eq. (5). With heavier objects such as basketballs (masses between 567 and 624 grams), increases were appreciable. With lighter objects such as ping pong balls (masses around 2.7 grams), the corresponding increases were very, very small. With tennis balls, objects with weights between those of basketballs and ping pong balls (masses of tennis balls are between 56 and 59 grams), however, increases in the duration of impact were moderate but still measurable with the techniques that were used in these experiments.

6. Conclusions

What was learned in Dynamics was applied to the bouncing of three types of game balls: ping pong balls, tennis balls, and basketballs. The projects involved analytical and experimental investigations of the relationship between the duration of a linear impulse and the energy dissipated during impact. Analytical results were supported by experimental data [15, 20, 24].

In summary, for each of the tested balls, analytical results and experimental data showed that, when the duration of impact increased, so did the amount of energy that was dissipated. Similarly, when the duration of impact decreased, so did the amount of energy that was dissipated [15,24]. Consequently, for each tested ball, the longer the duration of the impulse, the more energy was dissipated.

7. Impact on learning

It is reasonable to ask what the impact of this series of Dynamics projects had on student learning. The answer to this question has three parts. Part one has to do with specific learning outcomes, part two with what students learned to do during the projects, and part three has to do with what they gained in the process.

Part 1. Specific learning outcomes are mixed: quizzes and exams that covered central impact and the conservation of energy yielded very good results in that more than 90% of the class could solve the corresponding problems correctly. However, this knowledge acquired in central impact did not transfer to problems involving oblique impact in that only about 40- 57% of the class could solve problems involving oblique impact correctly.

Part 2. What students learned to do. They learned to [24]:

- Create a model for a real bouncing ball using particle mechanics;
- Apply the use of the conservation of energy in the analysis of a bouncing ball;
- Apply the use of the conservation of linear momentum in the analysis of a bouncing ball;
- Apply central impact, inelastic impact, and the coefficient of restitution to a real problem;
- Design experiments;
- Carry out their experiments and collect data using new software found on the web;
- Interpret data and relate results to what analysis had led them to expect;
- Write reports;
- Present reports orally; and
- Work in group.

Part 3. What students gained [24]. They:

- Engaged another dimension of learning by working on a hands-on project;
- Discovered that, even though the project required a lot of time and energy, the project was fun and more popular than taking an exam. Indeed, when given a choice between an exam and a project of equal weights, students overwhelmingly choose to do a project;
- Had some control over what they did, how they did it, and when they did it;
- (Those who started work early) discovered that they could do things over and ask for help, if/when things did not work well the first time;
- Had ample time do the work in and could pace themselves;
- Experienced working in groups with their peers [25];
- Could divide work among group members and share experiences, skills, and knowledge[25];
- Had something practical to talk about with their friends who are not studying engineering; and
- They found a subculture that provides opportunities for support and commiseration[25].

Matusovich, Streveler and Miller reported the results of their research on why students choose engineering[26-27]. Their work was focused on the subjective task value (STV) construct of Eccles, which is based upon the observation that an individual assigns a personal importance to engaging in an activity. Their salient conclusion is that many students choose engineering because they believe that it is consistent with their sense of self. However, in order to persist in engineering, that belief must be reinforced by the student's personal experience of what engineering is. Accordingly, it appears that the choice of whether or not to persist in engineering is not a decision that is made once and forgotten. Rather, it appears to be one that students revisit continually. Accordingly, the authors recommend that, given the diversity of students in engineering, instructors need to give students many examples of ways in which engineering is practiced. Projects in a variety of classes serve that purpose.

Finally, in an article on adding value to teaching, Chachra asked the following question: “what can we offer that students can’t get online? [28]” She suggested these three things: “Membership in a learning community, individualized mentorship, and hands-on practice (including access to scientific and engineering equipment)”. A project such as the one described in this paper adds all three things to teaching[29].

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