



## **Unlocking the Gate to Calculus Success: Pre-Calculus for Engineers - An Assertive Approach to Readyng Underprepared Students**

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# **Unlocking the Gate to Calculus Success: Pre-Calculus for Engineers—An Assertive Approach to Readyng Underprepared Students**

## **Abstract**

In general, underrepresented engineering students who enter the college underprepared in mathematics lack the basic skills necessary to succeed in calculus. Numerous factors contribute to these issues, including high school mathematics instruction deficiencies such as the absence of qualified teacher resources, poor instruction, and low student interest in higher-level math. The Engineering GoldShirt Program at the College of Engineering and Applied Science at the University of Colorado Boulder is piloting a well-supported new *pre-calculus for engineers* course in the college with the aim to adequately prepare these students for success in calculus, so as to not lose them at the gateway of the engineering education pathway.

In previous semesters in an effort to gain pre-calculus mastery, these GoldShirt students took math courses outside of the college in order to prepare to take the calculus sequence in the engineering curriculum. A subsequent review of these students' results in calculus I showed less than stellar performance. Of the students who took math courses outside of engineering, 64% earned less than a B- and 33% received Ds or Fs in calculus I. In response to this failure at the important mathematics juncture to success in engineering, a new pre-calculus for engineers course was developed in collaboration with the Applied Math Program to prepare students with a deeper understanding of mathematical concepts beyond what they previously received and prepare them for calculus success. A pilot class of 16 took the course in spring 2012, of which many moved on to calculus I in fall 2012 (pilot 1). A description and modifications to the pre-calculus for engineers course are presented in the paper, including the adoption of the ALEKS Learning System to assess and teach student math skills. Students are expected to take ownership of their learning and progress through modules to attain concept proficiency while meeting the lecture and recitation expectations for the course. Student performance in ALEKS contributes substantially to final course grades.

In fall 2012, pilot 2 of the class enrolled 29 students of which 22 were GoldShirt students, and 7 GoldShirt students enrolled in the course in the spring 2013. Two sections of pre-calculus for engineers will be offered in the fall 2013 semester (pilot 3). Other first-year students in the college placed below the threshold for calculus I readiness, based on placement scores from ALEKS, and are not included in the new course (about 100 additional students or 14% of the

new first-year class). Recommendations made to the college to expand the pre-calculus for engineers course have not yet been adopted.

This paper examines the performance results of the pre-calculus for engineers course and compares them to students' subsequent calculus I performance, uncovering the keys to proficiency in math and calculus success for engineering students with high potential who are underprepared in mathematics. To gauge student understanding and mastery of foundational mathematical concepts necessary for calculus success, the results of course assessment via post-survey, focus group and case study are described.

## **Introduction**

Although calculus I is a traditional entry point for first-year engineering students, for a variety of reasons the course generates a high failure rate. Poor performance in this “gateway” course no doubt leads many students to reexamine their decisions to study engineering. Compounding the scenario are widespread efforts to diversify the student population of engineering with the very students who are statistically most likely to graduate high school underprepared for direct entry into calculus I—minorities, women and first-generation college attendees. Thus, achieving diversity in engineering is linked to the performance outcomes in mathematics of students historically underrepresented in engineering, including the underrepresented minority (URM) populations of Black, Hispanic, Native American and Pacific Islander ethnicities. Currently U.S. engineering colleges are pervasively making efforts to broaden participation to include students from these backgrounds. Consequently, many institutions have looked to the mathematics entry point as a critical factor to control in their efforts to improve student performance, retention and diversity in engineering.

For many years, colleges and universities have used some type of measure to guide students into mathematics courses that best match their preparation levels. Most often, the calculus readiness baseline measures are standardized test scores, such as the ACT and SAT math scores that are commonly used for admission. Some argue that recruiting students with higher ACT math scores—in the range of at least 26—qualifies them to be enrolled in calculus I<sup>1</sup>. ACT scores are found to have a degree of correlation to success in calculus I<sup>2,3</sup>. However, the prediction of calculus I success is less strongly correlated to ACT scores than a combination of algebra skills and high school rank<sup>4</sup>. Many institutions give credit for students' high school calculus accomplishments when AP exam scores are earned of at least 3 or 4 (varies by institution), and higher AP scores correlate to better grades in the first semester calculus course<sup>5</sup>. Greater success was also noted for students who took yearlong calculus courses in high school, even if not AP-based<sup>6</sup>. A relationship exists between the highest levels of math course completed in high school to college course placement<sup>7</sup>. Calculus readiness is correlated to high school GPA<sup>8</sup>, which suggests that most engineering students, who tend to be near the top of their high school classes,

should be successful in calculus I. Yet an overall, high failure rate for calculus I takers persists; more than 35%-40% of students do not pass the course<sup>6,9,10</sup>.

Some institutions employ the additional approach of various in-house math assessment tools to guide students into the appropriate math courses. While in the past, students took these assessment tests during orientation and/or the first week of classes, now students submit assessment responses online, enabling them to obtain earlier guidance for enrollment into the most suitable math course (or remediation steps to take). Purdue University's Mathematics Science Inventory assessment relies on self-reporting to determine a student's level of familiarity and confidence in various math topics. The tool has been shown to be effective in helping to place students, as well as providing information to guide students toward academic support resources<sup>11</sup>. At the New Jersey Institute of Technology, assessments to determine algebra proficiency have helped to optimize course placement<sup>12</sup>. By using the Math Inventory in combination with standardized test scores and high school class rank to place students, the University of Pittsburgh reported that 94% of the students enrolled in calculus I performed satisfactorily or better; it also indicated a need for twice as many students to take pre-calculus compared to the prior year<sup>13</sup>.

Numerous strategies to boost students' mathematics skill levels have been documented. Some engineering colleges implement math improvement curricula into their summer bridge experiences or summer courses<sup>14,15,16</sup>. Evidence suggests that participation in math refresher activities boosts math assessment test scores, thereby enabling students to avoid remediation courses—calculus preparatory courses such as pre-calculus and algebra—and subsequently gain entry into higher level math courses such as calculus I<sup>17,18</sup>. A novel intervention to identify struggling students who withdrew midway through the semester by enrolling them into a mid-semester calculus preparation course helped students fill in knowledge gaps and better prepare for calculus in the subsequent semester<sup>19</sup>. Peer tutoring has been demonstrated to improve student performance in math courses<sup>20,21</sup>. In recent years, online tutorial systems have been implemented to improve student learning<sup>10,22,23,24</sup>.

One commercially available online mathematics tutorial product has been implemented by many institutions as an effective placement and remediation tool. The Assessment and Learning in Knowledge Spaces (ALEKS) system is a web-based, artificially intelligent mathematics learning product that institutions implement for assessment and that students use to improve their knowledge and facility with pre-calculus topics<sup>25</sup>. In combination with ACT math scores, the ALEKS assessment scores (from 0-100) show correlation to correct placement and subsequent success in calculus I<sup>26</sup>. Use of ALEKS as an assessment tool is increasing, and some institutions require specific ALEKS scores for entry into pre-calculus (at least 40) or calculus I (at least 65)<sup>27, 1</sup>. At Boise State and University of Illinois, incentives for students to enroll into courses that match their abilities were based on attaining defined threshold ALEKS scores, proof of

which counted for 10% of the course grade<sup>27,28</sup>. Institutions also use ALEKS as a self-paced learning method to help students improve their knowledge and scores so as to qualify for higher level courses<sup>20</sup>. The ALEKS online learning environment supports students in courses from pre-calculus through calculus II, either for homework and self-testing, or to self-remediate baseline concepts that enable better understanding of more advanced topics<sup>28,27,26,23</sup>. The use of ALEKS shows positive correlations to success in calculus I<sup>22</sup>. Benefits to using the system include easy website access, individualized learning plans, visual feedback of one's mastery level of topic areas, and the ability to work at one's own pace<sup>23,29</sup>.

### **Math Assessment and Preparation at University of Colorado Boulder**

The Applied Math program is responsible for teaching the four-semester course sequence, calculus through differential equations, at the College of Engineering and Applied Science at the University of Colorado Boulder. Program instructors also teach more advanced math courses for students who major or minor in applied math. Incoming first-year engineering students take a required placement test, and their scores direct them to course options<sup>30</sup>. An in-house assessment test was used until 2012 when the campus decided to administer all incoming first-year students, including engineering majors, the ALEKS assessment. In the calculus I and II courses, instructors now give the in-house assessment test—primarily a test of algebra competence, shown to correlate well to success in calculus I—during the first week of the semester to confirm correct course level placement. About 20% of incoming first-year students are guided to enroll in a two-semester calculus I + algebra course sequence, including a number who originally selected the single-semester calculus I course. Other students join the two-semester course after a poor showing on the first midterm exam. On average, the two-semester calculus I students outperform their one-semester counterparts in the subsequent calculus II course.

Figure 1 shows the ALEKS score distribution for the 735 students who were directly admitted into the fall 2012 first-year class. Using this data, the college recommended that 84% of the entering class (617 students who attained ALEKS scores  $\geq 65$ ), enroll in calculus I. Another 13% (96 students) scored in the range recommended for the two-semester, calculus I sequence. Three percent (22 students) were recommended to enroll in pre-calculus. Because the college recommendations are guidelines rather than rules, students are able to choose courses for which they may not be prepared to take. For example, 52 of the 96 students advised to take the two-semester calculus I actually enrolled in the one-semester calculus I. And, of the 22 students whose ALEKS scores ( $\leq 44$ ) directed them to take pre-calculus, nine students enrolled in the one-semester calculus I, going against college recommendations. The college has found that the mixed messages that students receive from their high schools, standardized test performances, advisors and parents lead a number of students to choose more difficult courses for which they are underprepared. Note that students must take the pre-calculus course in the College of Arts and Sciences, since it is not offered through College of Engineering.

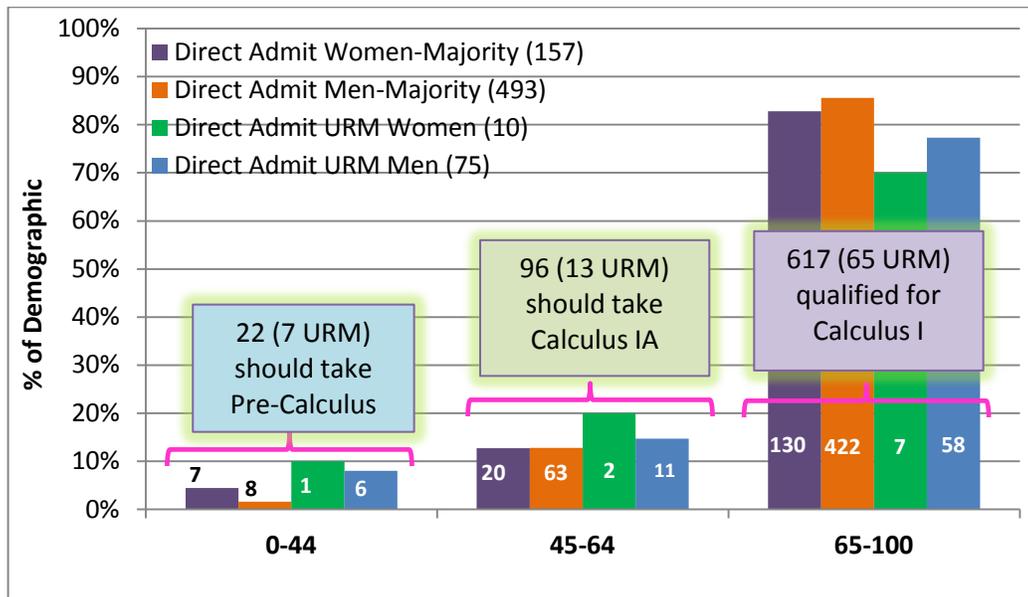


Figure 1. ALEKS score distribution by demographic for all students directly admitted into the College of Engineering.

It is notable that 20 out of 85 (24%) URM students are not ready for calculus I; likewise, 30 out of 160 (19%) women (including 3 URM women) are not ready for calculus I. When viewed through this lens, 47 out of 242 women and URM students (19%)—students who are key to increasing diversity in the college—are not prepared for success in calculus I.

### Engineering GoldShirt Program Overview

The College of Engineering is in the fourth implementation year of a diversifying program called the Engineering GoldShirt Program with the goal to enroll and graduate “next-tier” students with high potential but poor high school academic preparation. Each cohort of 32-36 students was chosen after a day of testing, interviews and observation in team settings. The program’s goals are to provide expanded opportunity and a *performance-enhancing “Engineering GoldShirt” year* for motivated high school graduates who are not *yet* fully prepared to succeed in an undergraduate engineering program, as an avenue for the college to increase enrollment and retention of students historically underrepresented in engineering—minorities, women and first-generation college attendees. The Engineering GoldShirt Program strives to build community while providing academic scaffolding to enable students to complete engineering degrees and achieve excellence as they do so.

The program admits disadvantaged, under-prepared high school graduates who demonstrate potential to be successful engineers, but fall short of meeting standard admissions criteria. The selected students are awarded scholarships during each year of the program that amount to about

one-third of the college tuition cost, and are expected to complete BS degrees in five years, including the Engineering GoldShirt Program first year. Various elements of the program are designed to build community, ignite excitement about engineering, and prepare students to succeed in engineering. Entering engineering GoldShirt students participate in a two-week summer bridge program to orient them to the challenges of college, building community among their peers, and developing leadership skills through a wide range of activities. During the initial year, students learn in small, cohort-based classes in mathematics, introductory physics, chemistry, writing and critical thinking. These students are placed into appropriate mathematics classes based on an in-depth review of placement exam results and high school transcripts, with the intent to ensure they are prepared to enter or have begun the engineering calculus sequence at the close of their first year. Students who achieve predefined metrics in the first GoldShirt year continue on with the standard curriculum for their engineering major, along with ongoing required participation in community-building and service-learning activities throughout their subsequent four years. This high-touch program is housed with the BOLD Center’s academic program and supported by other college and campus services and programs<sup>31</sup>. The BOLD Center houses the diversity program in the college.

### Engineering GoldShirt Pre-Calculus Background

The ALEKS scores of the Engineering GoldShirt students cover the entire range; the distribution of scores for the fall 2012 cohort is shown in Figure 2.

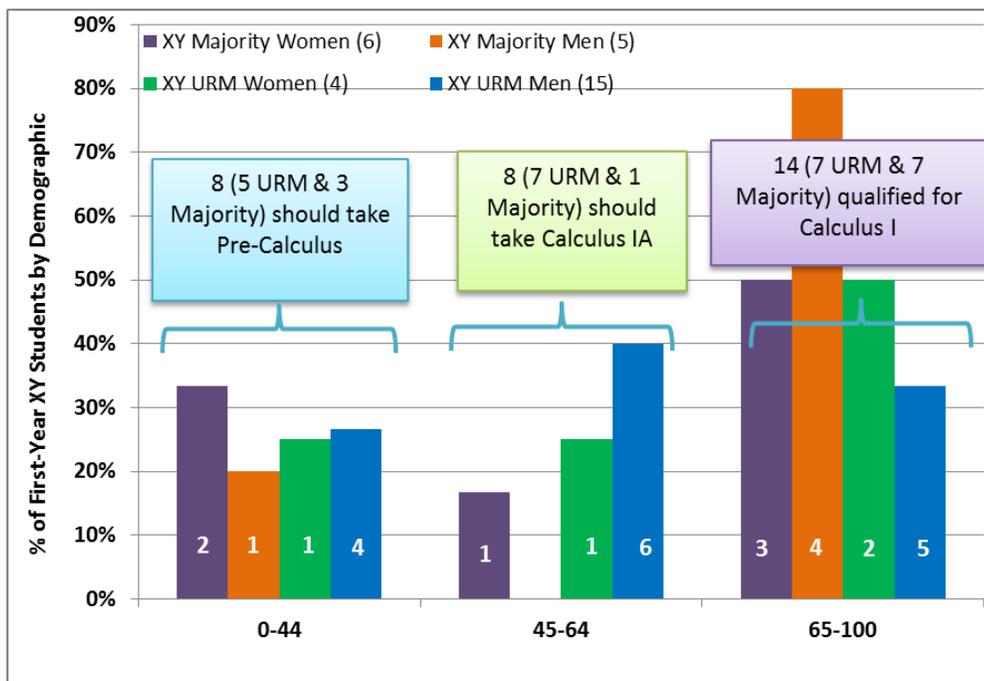


Figure 2. ALEKS score distribution by demographic for fall 2012 entering Engineering GoldShirt Program students.

Forty-seven percent of the cohort scored at least 65, 26.7% scored in the middle range of 45-64, and 26.7% scored below 45. These results support the suggestion that Engineering GoldShirt students on average are less math-accomplished than the directly admitted students. A pre-semester survey of these students indicated less confidence and preparation in mathematics, leading to a more conservative math course placement.

In previous semesters in an effort to gain pre-calculus mastery, some earlier cohort Engineering GoldShirt students took College of Arts and Sciences math courses, including trigonometry and pre-calculus, to prepare for the calculus sequence in the engineering curriculum. Generally, these GoldShirt students received As and Bs in these courses so it was assumed that they had achieved pre-calculus competence. However, a review of their subsequent performances in calculus I showed poor overall success—64% of these students earned grades of less than B-, and 33% received Ds or Fs in calculus I. It became apparent that pre-calculus needed to be taught to these students inside the college, since the outcomes from the courses taught outside the college were not aligned with the expectations and rigor of the engineering-based calculus courses. In response, a “pre-calculus for engineers” course was developed in collaboration with the Applied Mathematics program. Providing the pre-calculus course within the college has the benefit of serving as another induction point to build GoldShirt students’ sense of belonging in the college.

In fall 2012, 70% of the Engineering GoldShirt cohort enrolled into pre-calculus, 20% into two-semester calculus I, and only 10% into calculus I. After an initial offering (pilot 1 in spring 2012) of pre-calculus *for engineers* (taught without the support of ALEKS learning system components) demonstrated improved gains in math outcomes, the current cohort of Engineering GoldShirt students were enrolled into a revised, ALEKS-infused pre-calculus for engineers course (pilot 2). This paper examines the performance outcomes of the Engineering GoldShirt students enrolled in both pilot courses of pre-calculus.

### **Pre-Calculus for Engineers—Course Description**

The pre-calculus for engineers course addresses the needs of students who have not mastered concepts in algebra, trigonometry, and selected topics in analytical geometry so that they are prepared to begin calculus courses for engineers. The course objectives are to prepare students to have a deeper understanding of mathematical concepts beyond what they have received previously, in order to lay a foundation for success in calculus I and beyond. It requires students to engage in diligent work sessions as they learn the concepts. It is structured to acquaint students to the pace and culture of learning encountered in engineering and engineering math courses. During the course, through three lectures and one recitation per week, students learn to master the topics listed in Table 1. The required text for the course is *Precalculus: Mathematics for Calculus*, 6th edition, by James Stewart, Lothar Redlin, and Saleem Watson; ISBN10: 0840068077.

*Table 1. Pre-calculus for engineers course: units, topics and timeframe.*

Unit	Topics
<b>Algebra Fundamentals (12 hours)</b>	Real numbers, exponents and radicals, algebraic expressions
	Rational expressions, equations, modeling with equations
	Inequalities, coordinate geometry, optional: graphing calculators; solving equations and inequalities graphically, lines, making models using variation
<b>Functions (16 hours)</b>	Defining a function, graphs of functions Getting information from the graph of a function
	Average rate of change of functions, transformations of functions, combining functions
	One-to-one functions and their inverses, quadratic functions and models, polynomial functions and their graphs
	Dividing polynomials, real zeros of polynomials, complex numbers, complex zeros and the fundamental theorem of algebra
<b>Exponential Functions and Logarithms (12 hours)</b>	Exponential functions, the natural exponential function, logarithmic functions
	Laws of logarithms, exponential and logarithmic equations
	Modeling with exponential and logarithmic functions
<b>Trigonometry (20 hours)</b>	The unit circle, trigonometric functions of real numbers, trigonometric graphs
	More trigonometric graphs, inverse trigonometric functions and their graphs, modeling harmonic motion
	Angle measure, trigonometry of right triangles, trigonometric functions of angles
	Inverse trigonometric functions and right triangles, the law of sines, the law of cosines, trigonometric identities
	Addition and subtraction formulas, double-angle, half-angle, and product-sum formulas, basic trigonometric equations

The grading criteria for the course were established as follows:

- *15% ALEKS assignments (online homework)*—Assigned and matched with the course content and pace. Students were required to complete certain module assignments by deadlines to receive full credit.
- *10% Paper Homework*—Students were required to submit handwritten assignments that demonstrated their ability to solve problems step by step. Students were graded on the accuracy of their work, as well as their reasoning and problem solving ability.
- *10% WebAssign Homework (online homework)*—Students were required to submit WebAssign homework, another online assignment. This is the same online homework system used in the calculus for engineers courses and is tightly coupled with the course textbook.
- *45% Midterms (3 midterms worth 15% each)*—Midterms were administered in two parts: One part to assess student mastery of basic concepts and the other part to assess student problem solving ability via more open-ended and challenging problems.
- *20% Final Exam*—A comprehensive final exam included concepts from all course units.

### *Using ALEKS in Pre-Calculus*

Incorporation of the ALEKS system enables instructors and students to assess math skills in a variety of categories and for students to learn specific skills in areas that they are not proficient. Students are expected to take ownership of their learning and progress through the modules to improve their understanding of concepts, while also meeting other course expectations. The following ALEKS modules were included in the pre-calculus for engineers course:

- Algebra and Geometry Review (87 topics)
- Functions and Graphs (55 topics)
- Polynomial and Rational Functions (36 topics)
- Exponential and Logarithmic Functions (20 topics)
- Trigonometry (36 topics)
- Conic Sections (3 topics)

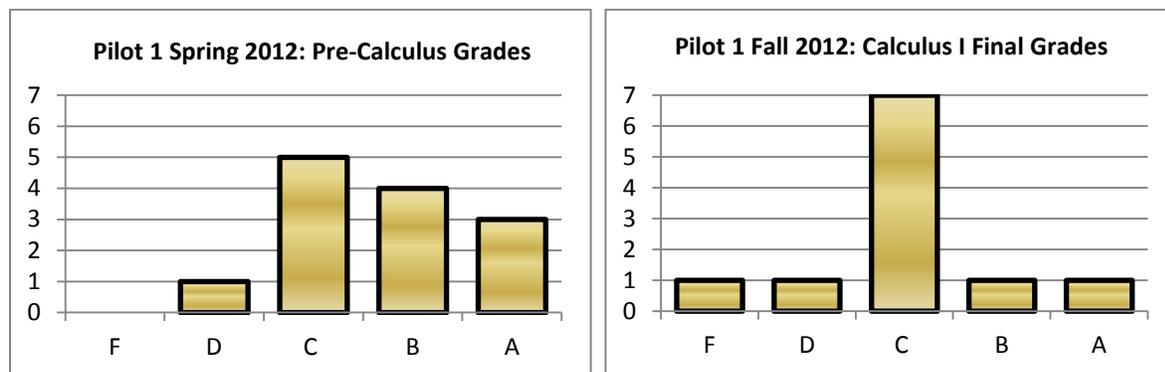
An initial assessment conducted early in the semester in ALEKS provides a baseline of what students have/have not mastered. This initial assessment was given over a two-week period that was work and time intensive. Several students did not take the ALEKS assignment seriously during the initial assessment and submitted answers to questions that may not have been well thought out, so ALEKS assigned them lessons and problems that they may not have needed. This required those students to invest even more time and effort on work that may not have been necessary. Also, some students had bigger gaps than others and were required to complete additional assignments to develop mastery in fundamental concepts. Once the assessment was completed and, depending on a student's mastery, ALEKS assigned individualized student work to assist students with mastering concepts.

Students were expected to work independently through each module and non-mastered topics until they became proficient in each topic. ALEKS tracked student performance throughout the semester so that students and instructors could observe progress. The sophisticated artificial intelligence algorithm of the ALEKS learning system uses the effort and answers that students input into the system to determine personalized paths of lessons, problems and assessments.

#### **Class Performance Results: Pilot 1**

During the spring 2012 semester, a pilot class of 16 took the pre-calculus for engineers course (pilot 1). Analysis of previous student performance results suggests that students who earn B- or better grades in current math courses are more likely to succeed in subsequent math courses. Thus, we required Engineering GoldShirt students to earn B- or better grades in the course or be required to repeat it. Of the 16 enrolled students, 13 were GoldShirt students. Of those 13 students, seven earned a B- or better in the course (54%).

The lower than B- grades earned by the remaining six students (46%) required them to repeat the pre-calculus for engineers course the following semester. Only two of these students re-enrolled in the course in the fall 2012 semester as required, and both earned a B in the fall 2012 semester. Of the remaining four students, three students enrolled in the two-semester calculus I course and one enrolled in the one-semester calculus course, going against the GoldShirt repeat requirement. None of the four earned a B- or better in calculus (three Cs in the yearlong course and one F in the one-semester course), with the consequences that they must all repeat calculus I in the spring 2013 semester.



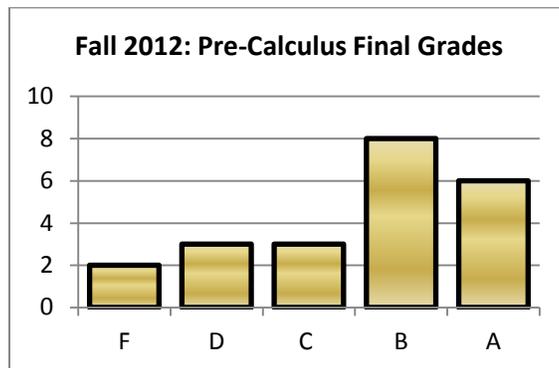
*Figure 3. Final grade distributions from the spring 2012 pilot 1 pre-calculus course and the subsequent fall 2012 calculus grade distributions.*

As this GoldShirt cohort moved on, of the 11 students who took calculus 1 in the fall 2012, only two students (18%) earned a B- or better—one student earned a B in one-semester calculus and the other earned an A in the first semester of yearlong calculus I. Nine students (82%) earned a C, C-, D or F. Compared to the previous results, a lower percentage of students earned Ds and Fs (33% compared to 18%). These grades are curved at the end of the semester with the mean set at B-/C+, and other letter grades are assigned at multiples of the standard deviation.

Though the pilot 1 course was designed to prepare students to excel in calculus, the mixed results suggest otherwise. Focus group results included student observations of much more emphasis on scoring well on the quizzes and exams and less emphasis on deep understanding, mastery and excellence. With pilot 1 results and feedback, the second pre-calculus for engineers course (pilot 2) was redesigned with the aim for students to “own” their learning, achieve deep conceptual understanding and excel at demonstrating their knowledge. These objectives were implemented through the incorporation of the ALEKS system, an instructor change, using learning assistants and teaching assistants, and pedagogical changes to lectures and recitations.

## Class Performance Results: Pilot 2

In pilot 2, 29 students enrolled in the pre-calculus for engineers course, of which 22 were Engineering GoldShirt students, including two repeaters from the spring 2012 course. Fourteen students (63.6%) passed with a B- or better. Of the eight students who did not pass with a B- or better, three (13.6%) earned a C or C-, three (13.6%) earned a D or D+ and two earned Fs (9%).



*Figure 4. Final grade distribution for the fall 2012 pre-calculus pilot 2.*

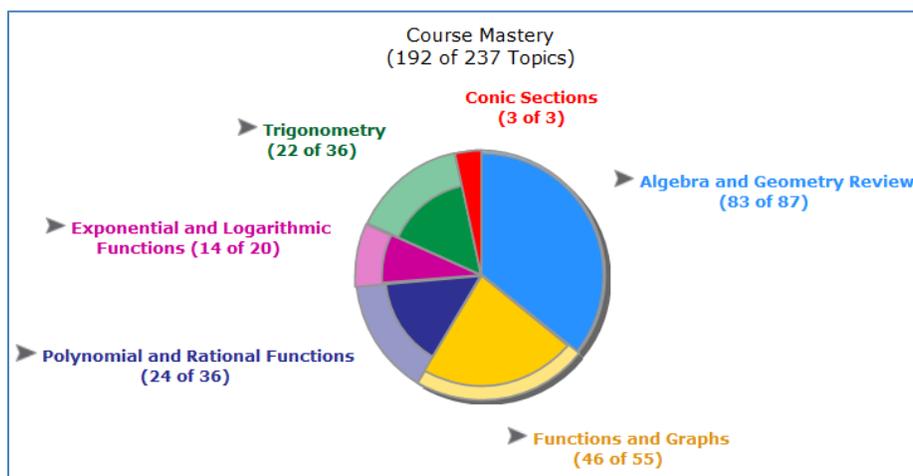
Of the 14 students who passed with a B- or better, six earned an A or A-, three of them are enrolled in one-semester calculus I, two are enrolled in yearlong calculus I and one left the program. Of the remaining eight students, six are enrolled in yearlong calculus I and two are enrolled in one semester calculus I. Their performances in calculus I will be evaluated at midterm and at spring 2013 semester end. Evaluation results will give a clearer indication of the preparedness of students who took the pilot 2 pre-calculus for engineers course.

The first round of spring 2013 exams in one-semester calculus I and yearlong calculus I for the 13 students is an early indicator of how students are performing in the course. For the one semester calculus I course, the college average was 71% on the first exam. The five students enrolled in one-semester calculus I scored, on average, 74.2% on the first exam, 5% higher than their peers in the college. For the yearlong calculus I course, the college average was 70% on the first exam. The eight students enrolled in yearlong calculus I scored, on average, 73.5% on the first exam, 5% higher than their peers in the college. While this data is preliminary and gains have not been tested for significance, early results are promising.

Of the eight students who did not pass the course with a B- or better, seven enrolled again in pre-calculus for engineers in spring 2013. Although these students mastered some of the concepts in the course, they still have knowledge gaps with major concepts necessary for calculus preparedness. Because of the small number of students, the same instructor who taught pilot 2 has implemented more active learning strategies, including oral assessment practice.

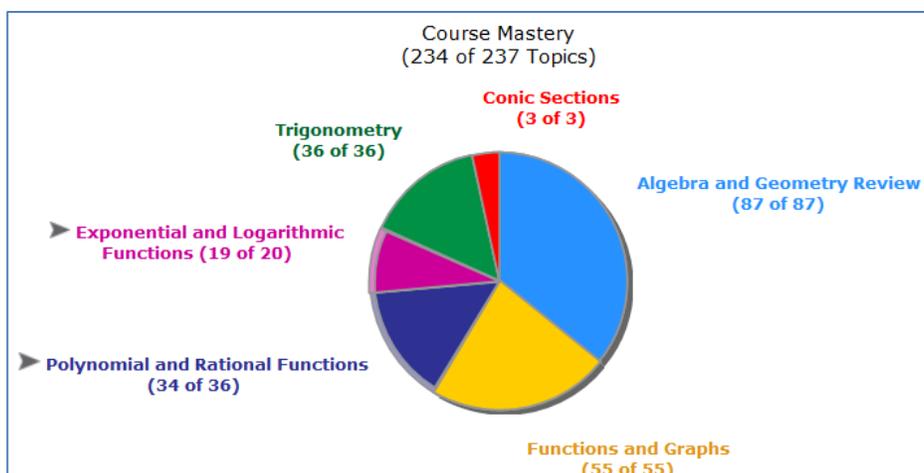
## Case Studies and Summary of Progress in ALEKS

The ALEKS learning system continuously tabulates and communicates student and class progress in a variety of ways, including progress (a list of objectives indicating what a student can do and what s/he is ready to learn), percent mastery since the last assessment, and a pie chart showing the state of overall course mastery. Each pie slice represents a topic module, and the mastery of each module is represented by the “filling up” of the slice. Complete module mastery is indicated by a completely full slice of pie, as demonstrated with the case study examples in Figures 6 and 7 that show ALEKS initial and final assessment pie charts. All students completed the ALEKS math placement assessment before the fall term; the initial and final ALEKS assessments were completed while completing ALEKS assignments during the fall term. The case study tracks the development of three students’ pre-calculus skills across the semester using output from the ALEKS mathematical software that students used throughout the course.



*Figure 5. Initial ALEKS assessment pie chart for student 1.*

For instance, in the Figure 5 example initial assessment, student 1 demonstrated strong competency in conic sections and algebra and geometry, and knowledge gaps in functions and graphs, polynomial and rational functions, exponential and logarithmic functions and trigonometry. Student 1 earned an initial ALEKS pre-semester, math placement score of 53/100 and an overall mastery level of 81% early in the semester when ALEKS was implemented. The ALEKS system calculated an average of 1.8 hours of week that student 1 dedicated to working on concepts in the system. Student 1 completed the semester with 99% mastery on the final assessment, as shown by the Figure 6 pie chart, which demonstrates this nearly perfect mastery of the ALEKS set of pre-calculus topics. Student 1 also performed very well in all other grading criteria categories, earning 90% or higher in all categories, shown in Table 2, and earned an A in the course.



*Figure 6. Final ALEKS assessment pie chart for student 1.*

*Table 2. Student 1's scores in the course grading criteria.*

Course Grading Criteria	Score
ALEKS homework	100%
Paper homework	98.2%
WebAssign homework	100%
Midterm exam average	94.7%
Final exam average	96%
<b>Final Grade</b>	<b>96.7% (A)</b>

Similar to student 1, student 2 earned a pre-semester, initial ALEKS math placement score of 52/100. Figures 7 and 8 illustrate the initial and final assessment pie charts of student 2 during the pre-calculus for engineers course. The ALEKS system calculated an average of 3.9 hours per week that student 2 dedicated to working on concepts in the system. The student began with 27% mastery on the initial assessment and ended the semester with 100% mastery on the final assessment. Student 2 earned above 90% in ALEKS and WebAssign homework, and earned 75-85% in paper homework and midterm exams, demonstrating somewhat inconsistent performance. These results are shown in Table 3. Student 2 earned a B in the course.

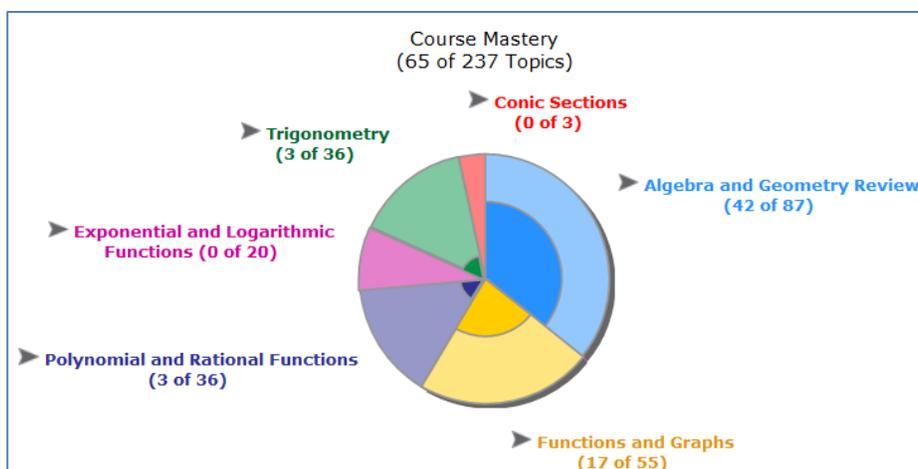


Figure 7. Initial ALEKS assessment pie chart for student 2.

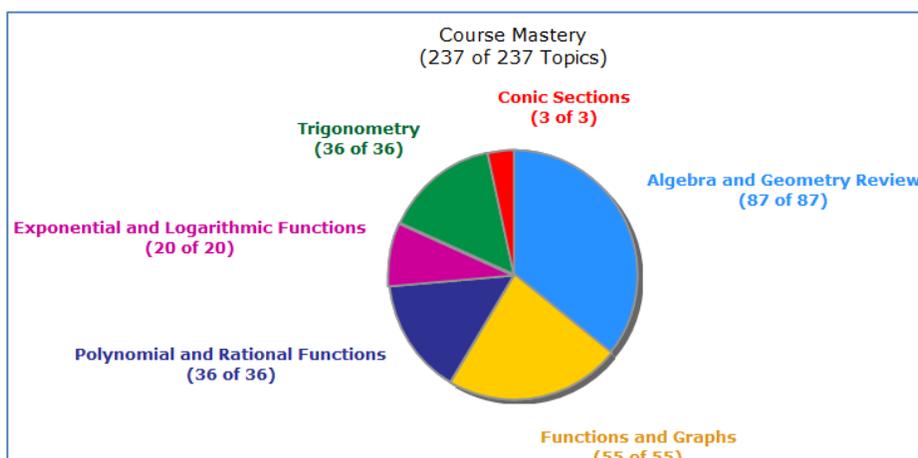


Figure 8. Final ALEKS assessment pie chart for student 2.

Table 3. Student 2's scores in the course grading criteria.

Course Grading Criteria	Score
ALEKS homework	100%
Paper homework	82.8%
WebAssign homework	99.5%
Midterm exam average	75.37%
Final exam average	84%
<b>Final Grade</b>	<b>83.87% (B)</b>

Student 3 earned a pre-semester, initial ALEKS math placement score of 18/100. The ALEKS system calculated an average of 4.1 hours of week that the student dedicated to working on concepts in the system. Figures 9 and 10 show student 3's ALEKS pie charts for the initial and final assessments that were ascertained in the pre-calculus course for engineers. The student began with 19% mastery on the initial assessment and ended the semester with 73% mastery on the final assessment. Table 4 shows the inconsistency of student 3's grades in ALEKS, paper and

WebAssign homework, including midterm and final exam scores that were below the class average. Instead of demonstrating strong work in the multiple course components, the inconsistent performance of student 3 resulted in a D+ course grade.

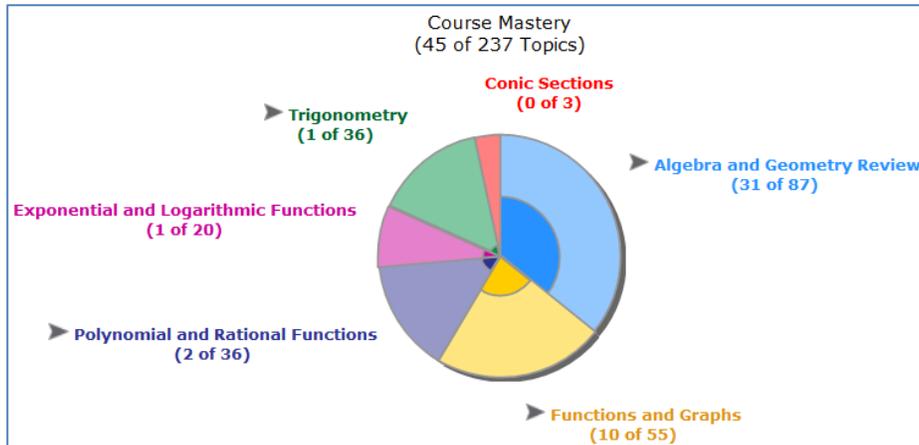


Figure 9. Initial ALEKS assessment pie chart for student 3.

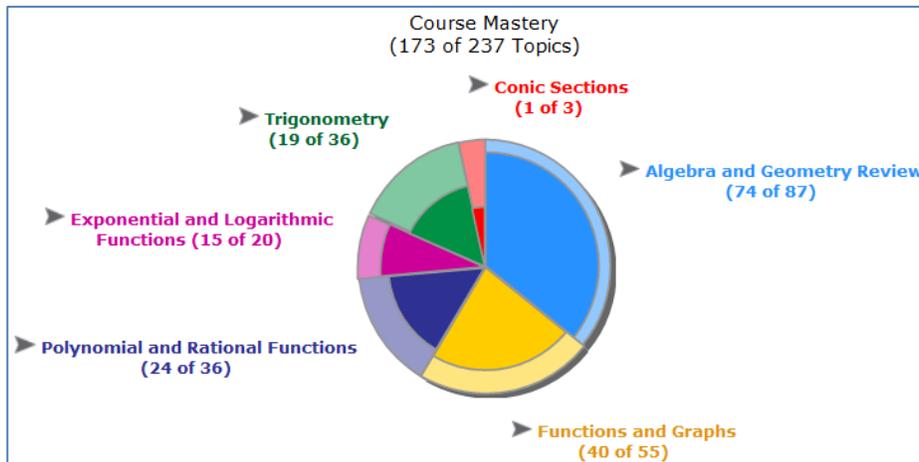
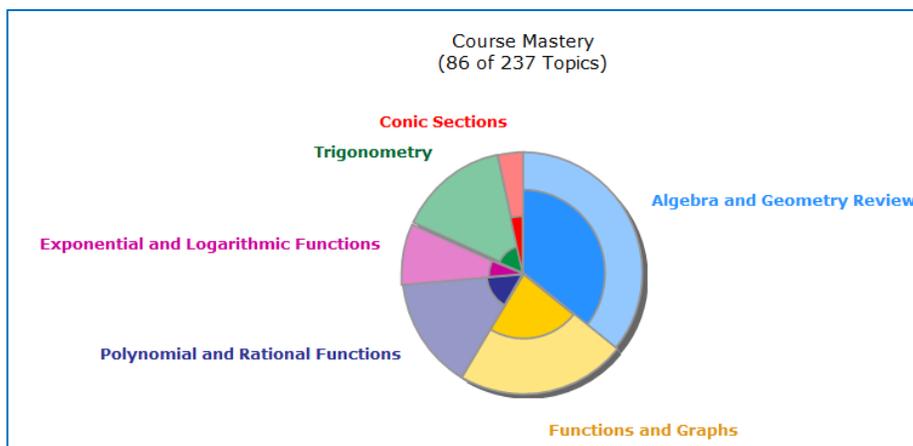


Figure 10. Final ALEKS assessment pie chart for student 3.

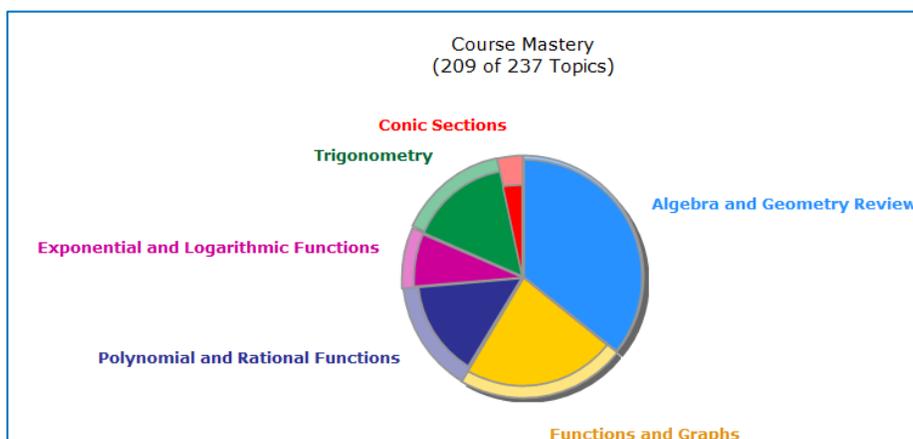
Table 4. Student 3's scores in the course grading criteria.

Course Grading Criteria	Score
ALEKS homework	75%
Paper homework	86.42%
WebAssign homework	91.51%
Midterm exam average	63.3%
Final exam average	58%
<b>Final Grade</b>	<b>69.05% (D+)</b>

Looking across the class performance as a whole, Figures 11 and 12 show ALEKS performance for the course.



*Figure 11. Initial ALEKS assessment pie chart for overall class performance.*



*Figure 12. Final ALEKS assessment pie chart for overall class performance*

Table 5 reflects the initial and post assessment results and percent increase for each topic. These results reflect significant growth for the class as a whole for all topics.

*Table 5. Class performance—mastery of ALEKS topics: initial and final assessment.*

ALEKS Objectives/Topics	Class Initial Assessment	Class Final Assessment	% Increase
Algebra and geometry review	58%	95%	64%
Functions and graphs	40%	86%	115%
Polynomial and rational functions	16%	82%	413%
Exponential and logarithmic functions	14%	83%	493%
Trigonometry	12%	86%	617%
Conic sections	18%	71%	294%

## Assessment Results

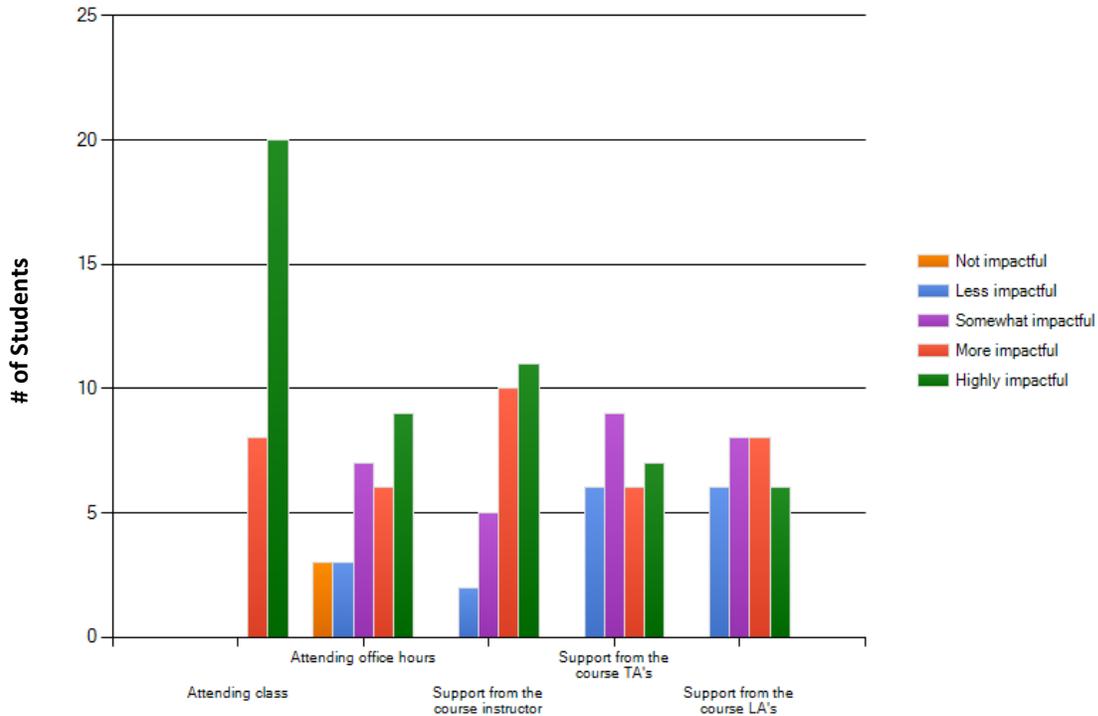
Data were collected from 29 participants in the pilot 2 pre-calculus for engineers course. The course included 28% women, 52% URM students, and 83% of students were in their first year of engineering. Course assessments included a post-survey, focus group and case study. The post-survey is composed of quantitative questions asking about the impact of the course on mastery of pre-calculus as well as experiences with course components. The focus group was conducted at the course end and had the class divide into teams that generated strengths and suggestions for improvement to the course.

The results of student self-ratings of their pre-calculus skills are listed in Table 6. While only 21.5% of students felt themselves to be moderately or highly skilled at the start of the course, 85.7% of students found themselves to be moderately or highly skilled at the end of the course, a gain of 298%. While the number of data points was too small to test for significance in this pilot course, gains in self-rated skills were substantial. The post self-ratings were significantly and positively correlated with course grades ( $r = .69, p < .05$ ) indicating that students were aware of their mastery levels in the class. The course grade was also positively correlated with ratings of the value of instructor support ( $r = .59, p < .05$ ) highlighting the value of the instructor in course mastery.

*Table 6. Self-rated mastery level of pre-calculus content (n = 29).*

Answer Options	Pre-Course	Post-Course
Not skilled	7.1%	0.0%
Less skilled	21.4%	10.7%
Somewhat skilled	50.0%	3.6%
Moderately skilled	17.9%	57.1%
Highly skilled	3.6%	28.6%

Student ratings of the impact of specific course components on their learning can be found in Figure 13. Students endorsed attending class as a key component for learning followed by support from the course instructor and attending office hours. While teaching assistant (TA) and learning assistant (LA) student support was viewed as less valuable, it should be noted that a percentage of students found these resources highly valuable as well. A TA is an Applied Mathematics graduate student, whereas, an LA is an undergraduate student. Both complete pedagogy training and additional experience with problem solving and how to explain concepts.



**Figure 13. Impact of course components on pre-calculus learning.**

Focus group results showed the course to be well-paced and comprehensive. Students like the small class size. The workload, not including ALEKS, was described as “about right” at 6-10 hours per week. However, students spent up to 20 hours per week on ALEKS assignments. The teacher was rated as enthusiastic and lectures were easy to follow. Suggestions for improvement were mainly around better incorporation of the online ALEKS software into the class. Students wanted milestone dates for module completion so that ALEKS work did not pile up on them. Students also requested that ALEKS topics better line up with weekly course topics. Use of this software was new, and students recognized that this was the first semester of its implementation and expressed hope for greater integration in the future.

Students completed a survey requesting feedback on the course. Several themes emerged from the following questions, along with illustrative student quotes:

**Question 1:** What did you like most about the pre-calculus for engineers course?

Emerging Themes: The instructor, the pace of the course and learning/preparing for calculus

*“One thing that I have liked about the pre-calculus course is that the professor would explain the material in depth and step by step. I liked the pace that the class has gone...”*

*“The way that everything we learned leads up to the next thing and is focused on the topics' applications to calculus. Also, the teacher has a good teaching style and works to have students understand the topics.”*

Question 2: What was most difficult about the pre-calculus course?

Emerging Themes: ALEKS, workload and pace

*“... the work load, it was a lot of homework to keep up with.”*

*“The ALEKS online assignments, because the large amounts of topics due at one time and also because when one makes a mistake in ALEKS, they have to do an extra similar problem, and sometimes these can compound.”*

*“I would say the class could improve by having weekly check-ups. Sometimes people learn at different paces and need to be checked on to see if they are correctly implementing the concept.”*

Question 3: Do you have any suggestions for improvement to the pre-calculus course?

Emerging Themes: Better integration and consistency of homework (WebAssign, ALEKS and written), remove ALEKS, offer more study guides, sessions and check-ups

*“I would choose between either ALEKS or WebAssign; having both not only is extremely stressful, but also confusing since both sources have different grading methods and teach differently.”*

*“My suggestions are to have review packets before a midterm. I noticed other math classes had reviews for the exams and they seemed to be very successful.”*

Question 4: Please describe your experience with the ALEKS instructional software.

Emerging Themes: Stressful, frustrating and confusing; did not like but learned a lot; enjoyed it and learned a lot

*“It's very frustrating, and I do hate it, but in the end it did teach me a lot and taught me a lot of new material.”*

*“I enjoyed it for the most part since, despite having its frustrating repetition thing for answering wrong, it actually helped me.”*

*“I did not like the software at all even though it does help, the set-up of how it works is not very fun since if you miss one you end up having to do more and more problems and becomes extremely frustrating.”*

## **Implications**

These results imply that strong performance on ALEKS assignments, paper homework and exams can support a student’s mastery of concepts, leading to student success in pre-calculus. Some students took advantage of the opportunity to take ownership of their learning: they worked hard using the tools available to them through this course and mastered the concepts. ALEKS also positively impacted course exam performance and supported their earning higher grades. Although some students progressed well in many of the ALEKS modules, additional time may be necessary if initial skill levels are too low to meet course expectations in one semester, requiring them to take another semester of pre-calculus.

Although student feedback indicated that students did not like ALEKS, they admitted that the tool helped them learn. Because ALEKS was implemented in the course a couple of weeks after the course began, students commented that it was unexpected and caused additional stress and that ALEKS assignments could have been better aligned and timed with the concepts taught in lectures.

Survey results indicate student awareness of their own concept mastery correlates to their performance in the course (final grade). Students were provided timely grading and feedback on assignments, which contributed to their awareness of ongoing performance in the course. Also, student feedback indicated the high effectiveness of instructors delivering comprehensive and easy-to-follow lectures. They appreciated that the course was well-paced, and that the instructor was competent, available, and had high expectations for them.

In observing class performance on WebAssign homework, little difference was seen in grades among students, so an assessment will be conducted to study the effectiveness of using this tool as required homework that represents 10% of the final grade. Students indicated the effectiveness of the WebAssign video tools, suggesting its potential use as a supplemental learning tool instead.

## **Future Work**

The spring 2013 pre-calculus course is designed to focus on the repeaters from the pilot 2 course who did not earn a B- or above. These students need a higher-touch, smaller, active learning environment and more time with the material. In oral sessions, students will be expected to

explain concepts and their problem solving strategies to the instructor. These students will also continue working in ALEKS to achieve mastery of all course modules.

Spring 2013 calculus I performance will be tracked for students who successfully completed the pilot 2 pre-calculus for engineers course. Their performance will indicate the effectiveness of the pilot 2 pre-calculus course, and help to guide changes necessary to improve the course for fall 2013 (pilot 3).

The ALEKS math placement test results indicate that a critical mass of freshman students enter the college who are not yet prepared for calculus I (see Figure 1). An initiative is underway to explore hosting multiple sections of the pre-calculus for engineers course to enroll these students who need calculus I preparation.

#### Bibliography

1. University of Texas Austin Cockerell School of Engineering, "Calculus Readiness Requirement," <http://www.engr.utexas.edu/undergraduate/admission/calculus>, 2012.
2. Wheeler, E., "Calculus Expectations and Retention," 2010 Proceedings of the ASEE North Central Sectional Conference, Pittsburgh, PA.
3. Jovanovic, N. S., "The Meaning of ACT Mathematics Scores: Implications for Engineering Programs, K-12 Schools, and Students," 2006 Proceedings of the ASEE Annual Conference, Chicago, IL
4. Edge, O. P., and Friedberg, S. H., "Factors affecting achievement in the first course in calculus." *The Journal of Experimental Educational* (1984), pp.136-140.
5. O'Connor, N. and Jones, G., "Success in Fall Math Course for Freshman Students Accepting AP tests/Villanova Equivalents for Introductory Math Courses—A Three-Year Study," 2010 Proceeding of ASEE Annual Conference, Louisville, KY.
6. Ferrini-Mundy, J. and Gaudard, M., "Secondary School Calculus: Preparation or Pitfall in the Study of College Calculus?" *Journal for Research in Mathematics Education* (1992): Vol. 23, No. 1, pp. 56-71.
7. Kauffman, P., Archava, S., Castles, R. T., Ries, H. L., Sullivan, S. T., De Urquidi, K. A., "Factors Impacting Performance in Pre Calculus," 2011 Proceedings of the ASEE Annual Conference, Vancouver, B.C.
8. Moses, L., Hall, C., Wuensch, K., De Urquidi, K., Kauffman, P., Swart, W., Duncan, S., Dixon, G., "Are Math Readiness and Personality Predictive of First-Year Retention in Engineering?" *The Journal of Psychology* (2011): Vol. 145, No. 3, pp 229-245.
9. Treisman, U., "Studying Students Studying Calculus: A Look at the Lives of Minority Mathematics Students in College," *The College Mathematics Journal*, Vol. 23, No. 5 (Nov. 1992), pp. 362-372.
10. Benson, L., Bowman, D., Hutchison, R., Wade, C., "Tutorials and In-Class Activity for Improving Student Performance in a First Year Engineering Course," 2009 Proceedings of the ASEE Annual Conference, Austin, TX.
11. LeBold, W. K., Budny, D. D. and Ward, S. K., "Using Student Self-Concepts in Placement and Evaluation," 1997 Proceedings of the ASEE Annual Conference, Milwaukee, WI.
12. Briller, V., Deess, P. and Zola, M., "Placement Tests as Predictors of Student Achievement in Mathematics, Chemistry and Humanities," 2002 Proceedings of the ASEE Annual Conference, Montreal, Quebec.

13. Shuman, L., Besterfield-Sacre, M., Budny, D., Larпкиattaworn, S., Muogboh, O., Provezis, S. and Wolfe, H., "What Do We Know about Our Entering Students and How Does It Impact upon Performance?" 2003 Proceedings of the ASEE Annual Conference, Nashville, TN.
14. Ruba Alkhasawneh, Rosalyn Hobson, "Pre-College Mathematics Preparation: Does It Work?" 2010 Proceedings of the ASEE Annual Conference, Louisville, KY.
15. Mary R. Anderson-Rowland, Maria A. Reyes, Mary Ann McCartney, "MEP Summer Bridge Program: Mathematics Assessment Strategies," 1998 Proceedings of the ASEE Annual Conference, Seattle, WA.
16. Pearson Weatherton, Y., Kruzic, A. P., Isbell, B. R., Peterson, L. L., Tiernan, J. C., Pham, V. V., "Mathematics Performance and First Year Retention of Students in Engineering Learning Communities," 2011 Proceedings of the ASEE Annual Conference, Vancouver, B.C.
17. Worley, D., Neubert, J. J., Kaabouch, N., Khavanin, M., "Leveling the Playing Field: Preparing Students for Calculus," 2012 Conference Proceedings of the ASEE Annual Conference, San Antonio TX.
18. Flores, B., C., Renner Martínez, J., Knaust, H., Darnell, A., Romo, L., and Kubo Della-Piana, C., "The Effectiveness of a Mathematics Review for Student Placement into College-Level Mathematics," 2003 Proceedings of the ASEE Annual Conference, Nashville, TN.
19. Robin Hensel, J. Ryan Sigler, Andrew Lowery, "The Cycle of Calculus Failure: Models of Early Math Intervention to Enhance Engineering Retention," 2008 Proceedings of the ASEE Annual Conference, Pittsburgh, PA.
20. Boerio, F. J., Roseman, R., Torsella, J., "Effect of Peer-Level Tutoring and ALEKS on Student Achievement in a Pre-Engineering Program," 2010 Proceedings of the ASEE Annual Conference, Louisville, KY.
21. Louie, B., Knight, D., and Sullivan, J., "A Drop-In Tutoring Program to Support First-Year Engineering," 2011 Proceedings of the ASEE Annual Conference, Vancouver, B.C.
22. Hampikian, J., Guarino, J., Chyung, S. Y., Gardner, J., Moll, A., Pyke, P., Schrader, C., "Benefits of a Tutorial Mathematics Program for Engineering Students Enrolled in Precalculus: A Template for Assessment," 2007 Proceedings of the ASEE Annual Conference, Honolulu, HI.
23. Carpenter, J., "Using Web-Based Technologies to Reach and Engage Millennial Students in Calculus," 2009 Proceedings of the ASEE Annual Conference, Austin, TX.
24. Worley, D., Neubert, J. J., Kaabouch, N., Khavanin, M. "Leveling the Playing Field: Preparing Students for Calculus," 2012 Proceedings of the ASEE Annual Conference, San Antonio, TX.
25. *ALEKS, Math for Colleges and Universities, General Information*, <http://www.highedmath.aleks.com/about/Welcome-ENGLISH.html>.
26. Carpenter, J. and Hanna, R. E., "Predicting Student Success in Calculus," 2007 Proceedings of the ASEE Annual Conference, Honolulu, HI.
27. Bullock, D., Callahan, J., Ban, Y., Ahlgren, A., Schrader, C., "The Implementation of an Online Mathematics Placement Exam and Its Effects on Student Success in Precalculus and Calculus," 2009 Proceedings of the ASEE Annual Conference, Austin, TX.
28. University of Illinois-Urbana Champaign, "Math Placement Score," <http://www.math.illinois.edu/ALEKS/score.html>, 2012.
29. Callahan, J., Chyung, S. Y., Guild, J., Clement, W., Guarino, J., Bullock, D., and Schrader, C. "Enhancing Precalculus Curricula with E-Learning: Implementation and Assessment," 2008 Proceedings ASEE Annual Conference, Pittsburgh, PA.
30. University of Colorado Boulder College of Engineering and Applied Science, "Math Assessment (ALEKS)," <http://www.colorado.edu/engineering/admissions/first-year/aleks>
31. Ennis, T., Milford, J., Myers, B., Sullivan, J., Knight, D., Sieber, D., Scarritt, A. "GoldShirt Transitional Program: Creating Engineering Capacity and Expanding Diversity through a Performance-Enhancing Year," 2010 Proceedings of the ASEE Annual Conference, Louisville, KY.