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Invited Paper - Virtual Environment: A Tool for Developing Students' Abilities to Apply Mathematics to Real-life Problems

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Virtual Environment: A Tool for Developing Students' Abilities to Apply Mathematics to Real-life Problems

Introduction

Long ago Freudenthal[1] wrote "The huge majority of students are not able to apply their mathematical classroom experiences, neither in the physics or chemistry school laboratory nor in the most trivial situations of daily life" (p. 5). Freudenthal believed that mathematics was needed not by a few people, but by everybody and as such it should be taught to be useful for everybody. He pointed out that the problem was not what kind of mathematics was taught, but how it was taught. Even the fact that the teacher could apply mathematics himself did not imply that he knew how to use his knowledge in his teaching. It often happened that mathematicians ignored aspects of reality when doing mathematics.

In recent research of 2007, Lesh & Zawojewski[2] posed the same problem and asserted that among mathematics educators there was a common recognition, that a serious mismatch existed and was growing between the skills obtained at schools and the kind of understanding and abilities that were needed for success beyond school. Almost at the same time Ilyenkov[3] wrote about a current problem of 'the practical application of knowledge to life'. Like Freudenthal in 1968, Ilyenkov in 2009 saw the problem in how the subject was taught. He asserted that the attempts of some instructional theories to solve the problem by creating systems of rules of 'how to apply knowledge to life' impeded rather than helped. Moreover, Ilyenkov[3] specified the source of the problem, saying that 'visual aid' provided to students created only an illusion of concreteness of understanding because it was created independent of the activity of the student. That is, the decisive part of cognition - to go from the object to an abstract - remained outside of student's activity.

The forty years old problem of teaching mathematics so that it would be connected with reality and useful for everybody appeared to be unresolved. "For all the talk about real-world mathematics, it seems like we still don't get it"[4].

Many teachers and textbook writers have been working on the development of mathematical school tasks that resemble out-of-school situations. Palm and Burman[5] reported that, in Finland and Sweden, in many of the tasks encountered by students in school mathematics the situation described in the task, was a situation from real-life. Such mathematical tasks, containing situations from real-life, have been traditionally described by words and commonly referred to the 'word problems'. Word problems are firmly entrenched as a classroom tradition, particularly in North American schools[6], and yet, there has been long lasting debates about the reasons for the lack of word problems' effectiveness as a link between abstract mathematics and real-life phenomena. Particularly, Gravemeijer[7] noted that research on word problems has revealed the complex nature of the processes that lead to the lack of students' activation of their real-world knowledge. Palm[8] stressed that in a large number of studies students did not pay much attention to the realities of the situations described in the word problems. Gerofsky[9], in turn, asserted that word problems were unable to be faithful simulation of real-life tasks. She insightfully predicted that there would appear new approaches based on new computer technologies.

The contemporary computer technologies undoubtedly can provide much better than word problems simulations of real world situations in mathematical classrooms for connecting the mathematical abstracts with out-of-school situations. The purpose of this study is to utilize contemporary technologies, and particularly Virtual Environment (VE), for bringing the physical world into classrooms so that the students would be able to apply their mathematical knowledge to the VE real-life problems and as such would develop their abilities to apply mathematics to beyond schooling situations.

Virtual Environment: A tool for simulation reality

The departure point in this study is the assumption that VE is a 'reality' for its users. In other words, VE is a contemporary technological tool which can 'bring the reality' into classrooms representing the physical world situations with high degree of fidelity and immersion. Massara, Ancarani, Costabile, Moirano, & Ricotta[10] claim that the immersion of the Second Life VE erases the difference between real and virtual worlds to the extent that, users' psycho-physical behaviors in VR becomes consistent with real life. Meredith, Hussain, & Griffiths[11] points out that, investigators consider the Second Life VE as a synthetic world. Many "residents" of the Second Life VE are escaping from their everyday real life into this synthetic world[12] which in turn means that the VE synthetic world becomes a reality for VE users. The term 'Virtual Environment' is also known and widely used as 'Virtual Reality' (VR), which reflects its essence of 'reality'.

Steuer[13] asserts that "presence" and "telepresence" are fundamental for definition of VE. Heim[14] identified the following concepts of virtual environment: *simulation, interaction, immersion, artificiality, telepresence, full-body immersion,* and *network communication*. Among all these concepts the first three, *simulation, interaction, immersion,* are the most important for this study goal of creating real-life in classrooms.

Any VE is a *simulation* of the reality which in turn allows creating a variety of real-life situations of different scales within a restricted space (e.g., a room, a computer screen). The majority of current virtual environments are based on visual displays presented either on a computer screen or through special stereoscopic equipment giving the perception of 3-D depth. In this research the choice of a computer screen VE is based on the fact that among five senses constituting human sensory perception (smell, sight, taste, touch, and hearing), the sight is the dominant one. It is believed that not less than 70 percent of all sensory information that is imported into humans brains comes from what we see[15,16]. There are a variety of technologies, algorithms and computer languages for simulations of physical reality on computer screens. For example, Campbell[17] mentioned 3D web-based graphic languages such as Virtual Reality Modeling Language (VRML), eXtensible 3D (X3D) graphics language; proprietary languages such as Java3D (a 3D extension of Java) and Shockwave 3D.

Such advances as 3-D stereoscopic equipment and equipment providing aural/haptic information should result in stronger feeling of presence in the environment, which in turn is commonly defined as subjective feeling of “being there” and mainly conceived as deriving from immersion, interaction, and social and narrative involvement with suitable technology[18]. Brooks[19] defined VE from the viewpoint of VE experience, reflecting both immersion and action. That is, VE shouldn’t be defined only by a number of technical advances; it should be connected directly with experiencing in it. Why presence in terms of immersion and action is so important for simulating ‘reality’ in mathematical classrooms? Immersion and action/interaction can provide students with the perception of the body action, which, in turn, creates an embodied sense of the action including such important calculus concepts as motion and velocity. Liljedahl[20] showed that it is unfavorable task to achieve perception and reflection of "motion and change" using traditional static methods of teaching and learning. Particularly, he described the difficulty of teaching motion in the absence of the embodiment of motion as well as the tension that was created between an embodied sense of motion and its static representations.

The concept of presence in VE can provide the perception of and reflection on properties of simulated real objects. In different VEs the body *immersion* can be achieved by different means. Campbell[17] noted,

> Indeed, in an important sense virtual environments in fact are quite real and embodied, but in different ways. First, those who are engaged in virtual worlds, such as SL for instance, are having *experiences* – real lived experiences. Secondly, they are embodied, albeit virtually, through their avatars. (p. 592).

One of the effective ways of immersion achievement is a choice of either egocentric or allocentric view perspectives, depending on individual preferences. According to Berthoz[21], the brain uses two frames of reference for representing the position of objects: egocentric and allocentric. For example, the relationships between objects in a room for estimation the distances and angles can be encoded either ‘egocentrically’, or ‘allocentrically’. In the first case, everything is related to yourself; a second way means of encoding spatial relationships between the objects themselves or in relationship to a frame of reference external to your body (ibid). That is, the egocentric perspective gives a perception of ‘being’ within the VE and seeing objects from the ‘first person’ view. The allocentric perspective is provided when an avatar is present in the environment and a learner controls the avatar navigation.

The VE concept of *interaction* allows students to manipulate and transform the simulated objects. The interactive function helps to overcome a major contradiction of traditional education - absence of correlation between schooled, passive knowledge and real object. According to Ilyenkov[3], one of the main reasons of absence of such correlation is that instead of a real-life object students are given a ‘ready – made’ image of it as a substitute with no activity with the object. As the result, the students encounter the object itself only outside of the school. Tall[22] wrote “It is possible to design enactive software to allow students to explore mathematical ideas with the dual role of being both immediately appealing to students and also providing foundational concepts on which the ideas can be built.” (p. 20). From the viewpoint of inactivity, VE is exactly the type of software Tall[22] wrote about in 1991.

Space is an additional background concept which is introduced in this study. Space connects VE, physical reality, and mathematics. It is a fundamental concept of geometry and calculus. Immersion, action, interaction – they all depend on our subjective perception of simulated space. Space and objects on computer screen of VEs have been modeled and programmed using different graphic languages. Campbell[17] showed that the majority of contemporary computer languages are designed for programming 3D space and 3D objects in space. Actually, even one of the earliest software package used for design VE objects, namely Computer-Aided Drafting (CAD) represented 3D objects in 3D Cartesian co-ordinate system. VEs with stereoscopic equipment, giving the perception of 3-D depth, provide better 3D perception.

For simulating a real-life problem for this particular research we chose the Second Life VE. Many recent publications are devoted to Second Life VE, to its popularity and application[10,11,12,17,23]. Second Life is an accessible (http://www.secondlife.com) and easy using VE. It has 3D computer graphics and high fidelity; it provides egocentric and allocentric view perspectives, becoming a reality for its users.

Realistic Mathematics Education

The realistic essence of VE and the aim of this study of utilizing VE for 'bringing reality into classroom' for application corresponding mathematical knowledge, determined the study theoretical framework. Realistic Mathematics Education (RME) is used as an appropriate theory for experimental design and data analysis. Particularly, the ways of students' application of mathematics to a simulated in the Second Life VE real-life optimal navigation problem are analysed through the lenses of mathematizing and modelling.

RME is a teaching and learning theory in mathematics education which is based on Freudenthal's idea that mathematics must be connected to reality. The use of realistic contexts became one of the determining characteristics of this approach. The subject matter should be experientially real to students[24]. In addition to being 'experientially real', the RME context implies that it also should be suitable for mathematizing, which in turn is a fundamental characteristic of RME. Freudenthal[1] wrote that, mathematics can best be learned by mathematizing. His primary focus was on mathematizing reality in the common sense meaning of the real world out there.

Treffers[25] formulated the idea of 'progressive mathematizing' as a sequence of two types of mathematical activity – horizontal mathematizing and vertical mathematizing. Horizontal one describes transforming a problem field into a mathematical problem. Treffers[25] suggests that horizontal mathematizing is constituted by nonmathematical problem field or related to real world situation. He affirmed that "in the horizontal component the way towards mathematics is paved via model formation schematizing, symbolizing" (ibid, p. 247). Vertical mathematizing is grounded on horizontal one and includes such activities as reasoning about abstracts, structures, generalization and formalizing within the mathematical system itself.

This study also uses the RME modeling principle for data examining. The important feature of RME models is that they should support progression in vertical mathematizing without blocking the way back. Another requirement for models in RME is that they can be re-invented by the students on their own and should be easily adapted to new situations. According to Streefland (cited in Van den Heuvel-Panhuizen[26]), models can fulfill the bridging function between the informal and the formal level: by shifting from a *'model-of'* to a *'model-for'*. At first, the model is a model-of a situation that is familiar to the students. By a process of generalizing and formalizing, the model eventually becomes an entity on its own. It becomes possible to use it as a model-for mathematical reasoning.

Finally, the RME guided reinvention principle is adopted and used as a part of methodology. According to RME instructional design theory, the teacher provides guidance, playing a 'proactive role' within the classroom setting. This study allows every student to decide whether and to what extent s/he needs guidance which in turn is provided in the specially designed guiding-reflecting journal.

Participants and Methods

Ten students from Vancouver Templeton Secondary School ranging in age from 17 to 18 years, 5 males and 5 females, participated in the study. They were at the end of AP calculus course and had completed such topics as application and computation of derivatives. The students were informed about the goal of the research and that the experiments would be conducted in the school's Teacher's room, outside of regular calculus class time and that each session would last 60-90 minutes.

The simulated in the Second Live VE setting includes a pond with shallow water, surrounded by bushes and trees (Figure 1).

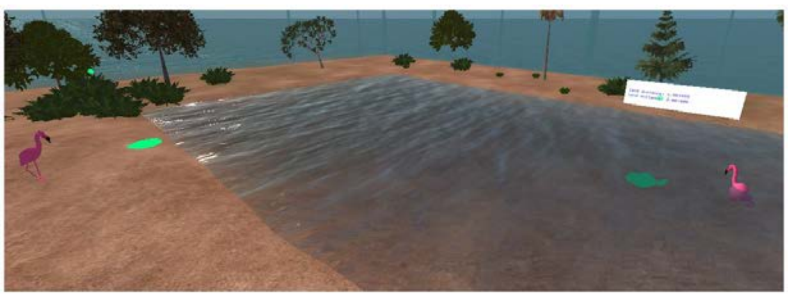


Figure 1. Simulated in the Second Life VE Setting

The environment is programmed so that walking/running speed on land is twice as fast as walking/running speed in water. There are two green small round platforms: one platform is located on land near the water's edge, another is located in the water (Figure 1). The optimal navigation VE Task is to find the path between the platforms which would minimize time of travel. The setting is programmed to record time and distance traveled by land for each trip between the platforms. This information is indicated on white banners, one of which is shown in (Figure 1). The SL allows utilizing both egocentric and allocentric view perspectives.

After each trip the student must transfer the data from the banners (time and distance traveled by land) into a specially designed guiding–reflecting journal, which is an integral methodological part of the research design.

The guiding–reflecting journal contains the instructions for VE activity followed by some information about water and land components of trips. The journal provides tables for 10 forward and return trips, then the student has to record the best (minimal) time out of these trips with corresponding value of distance traveled by land. Then the journal suggests the student to try to solve the problem mathematically with blank space for independent reasoning. At this point student chooses whether to develop his/her own model-of the situational problem or to accept the model offered in the journal.

The experimental design contains a few stages. The first stage is a so called 'exploration trial' as in (Mueller, Jackson, & Skelton[27]; Ross, Skelton, & Mueller[28]). This first stage is the students' free activities in the VE before they get the journal with instructions for the VE Task. The exploration trial is with unlimited duration until the student feels comfortable in the environment and announces that s/he is ready to start the next stage. This first stage allows students to explore the pond with its shallow water and to feel the speed difference on land and in water. The students have an opportunity to try egocentric and allocentric view perspectives and to choose the preferable one. Altogether, the goal of exploration trial is to let students get feeling of 'being' in the environment before starting the next, second, stage of the designed study which is the optimal navigation VE Task. At the beginning of the second stage students receive the guiding–reflecting journals with instructions. Their optimal navigation in VE is accompanied by working with the journal. The third stage of the designed study is mathematizing the VE activity which implies the journal work only. The final, forth, stage of the experimental design refers to completing the journal's questionnaire.

The data is drawn from 3 sources: video recording of students' mathematizing in the guiding-reflecting journals; screen capturing of their VE activities; guiding-reflecting journals.

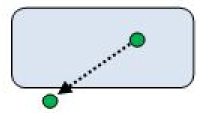
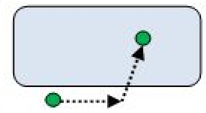
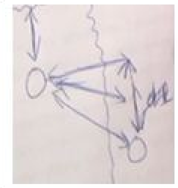
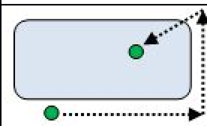
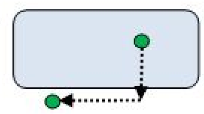
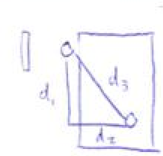
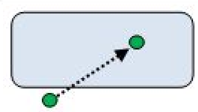
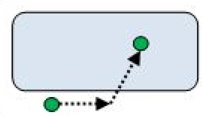
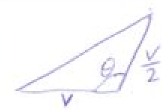
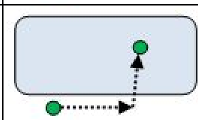
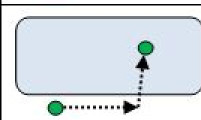
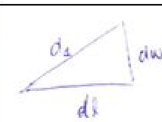
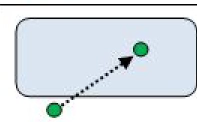
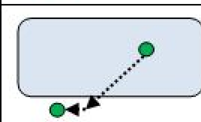
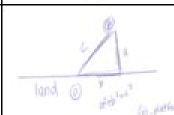
Results and discussions

The activities of five students, named Kenneth, Jason, Nick, Kate and Ann, were analysed. These five students were chosen out of ten participants because they performed five different ways of mathematizing which in turn allowed exploring the differences.

Table 1 bellow demonstrates data which were integrated so that to consider connections between the duration of exploration trial, computer game experience, and the first trip strategy. The last column of the table shows the students' graphical models-of the situational problem. This column allows seeing whether the best time trip strategy impacted the construction of the models-of the situational problem.

Analysis of the two first columns shows that the longer computer game experience, the less time students required for exploration trial. Particularly, Jason and Kate had the longest computer game experiences and spent the shortest time for the exploration trial. Their first trip strategy of increasing land distance was determined by the information that speed on land is faster than speed in water which they received from the journal. Kenneth, Nick, and Ann spent longer time for exploration trial, and navigated in both media; they also knew about speed difference from the journal but all of them chose the shortest distance between the platforms strategy.

Table 1. Integrated Data of Five Participants' Activities.

Name	Exploration trial duration	Computer games experience	First trip	Best time trip	Model-of the situational problem
Kenneth	3.45 min	1 hour per week			
Jason	30 sec	5 hours per week			
Nick	3.5 min	3 hours per week			
Kate	0	15 hours per week			
Ann	2 min	0 hour per week			

Comparison of the two last columns shows that the empirical best time trip strategies determined the graphical models-of the situational problem. Particularly, Kenneth's and Nick's best time trips were the trips of balanced land and water distances (with water distances between their minimal and maximal values) which, in turn allowed them to create models-of their empirical activity transferrable to model-for further mathematical development. Kenneth then applied calculus as the mathematical development of his model-of the situational problem. Nick used geometrical approach and was very persistent in developing his own model; at some point he preferred to accept and develop the journal model. All the other participants' best time trips were either maximum or minimum water distances which determined their right rectangle graphical model's-of their empirical activity and were not subjects to mathematical development for the reason of their stability and absence of variables to explore. They all accepted the journal model. Ann was the only student who was not able to follow the journal guidance. She needed additional detailed explanation of how to apply calculus to her VE activity. After such explanation she exclaimed, "Now it all makes sense!" meaning the material she was taught in her calculus course. This is an important point. This means that if students in the classroom are provided the opportunity to apply mathematics to their own VE activity, the learning material makes more sense to them.

The last two columns of Table 1 demonstrate the robustness of empirical knowledge obtained from activity in VE: in all five cases best time trips determined horizontal mathematizing of construction graphical modes-of the situational problem.

A background assumption which was made at the beginning of the research was that VE technology, and particularly the Second Life VE provides simulation on the computer screens close to the reality; and the real-life problems simulated in VE can be considered as the problems of real physical world. The fact that all five students, Kenneth, Jason, Nick, Kate, and Ann, developed their models-of the particular situational problem on the basis of their empirical activity in VE suggests that the Second Life VE indeed provides simulation close to reality. As such it can be used for developing students' abilities to apply mathematics to real-life problems.

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