CONNECTION THEORY AND SOFTWARE: EXPERIENCE WITH AN UNDERGRADUATE FINITE ELEMENT COURSE

Natasha Smith and Julian Davis
Department of Engineering
University of Southern Indiana

Abstract

Over the past several years, the commercial finite element analysis (FEA) software industry has seen significant growth in both capability and reach. As such, today's undergraduate students have access to sophisticated, yet easy to use simulation tools. For better or worse, use of the tools themselves requires neither an understanding of foundational principles nor a working knowledge of the finite element method. One could make the case that this is part of the natural evolution of any new tool (as one no longer needs to be a mechanic to drive a car). On the other hand, users absolutely need to know enough to understand the consequences of their own modeling choices (e.g. how boundary conditions are applied, element selection, mesh size, etc.). Thus, the proliferation of FEA software in industry (1) necessitates treatment of these tools at the undergraduate level and (2) suggests a balance be struck between the software and theory in these courses.

This paper details the authors' experience with a first course in finite element analysis within an undergraduate only engineering curriculum. In particular, the struggle to find the best balance between FEA theory and practical use of software is discussed. Within the course, students complete a variety of assignments using a mixture of resources that include hand calculations, Matlab by Mathworks, and DassaultSystèmes' SolidWorks. The course culminates in a self–selected student project requiring they assess the impact of modeling choices on results of particular interest.

One important finding is the limitations of some commercial packages in developing one dimensional models, an important stepping stone to understanding of FEA theory. In addition, the paper studies the impact of prior programming experience on a student's ability to succeed in the course. Finally, the authors have experimented with a course textbook which emphasizes use of software and alternatively, a text with more comprehensive treatment of FEA fundamentals.

Introduction

Finite Element Analysis (FEA) has its origins in the 1940s, as a numerical method to solve complex problems in engineering. In essence, FEA uses algebraic equations to approximate solutions to the differential equations which govern the physics of a wide variety of disciplines (e.g. elastic response, vibrations, fluid flow, heat transfer, etc.) These approximate solutions are applied to small 'finite elements' which are generated by discretizing a larger, complex system. Boundary conditions for the system, in addition to compatibility requirements between elements, provide the needed constraints to solve equations for all the elements in the system simultaneously. The development of computers led to growing acceptance of this method among the research community through the end of the twentieth century, as access to significant computational power paved the way to solve the algebraic equations for tens of thousands of elements and more. Logan provides a brief history of FEA development in the opening chapter to his introductory text on the subject [1].

Within the past 10-15 years, improvements in commercially available software, particularly the seamless linking between FEA and computer aided design (CAD), have led to more widespread adoption of the method as a standard tool of the engineering trade [2–4]. DassaultSystèmes', designer of SolidWorks software, suggests their products enable users to validate designs, ''without needing a Ph.D. in Finite Element Analysis," for example [5]. Bolstering this contention, recent program graduates are currently using FEA software for designing commercial vehicle wheels and structural polymer products. Meanwhile, undergraduates are often encouraged to use the method for capstone projects as well as intercollegiate design competitions [6,7].

The fact that FEA is now in the midst of the technological transition from a research tool to a professional competency has significance for engineering curricula at the undergraduate level. For one, FEA exposure to some degree is necessary to meet expectations of employers. Second, content of FEA instruction must adapt both to the availability of new tools as well as to a changing demographic of student. Dues made this case, arguing that the functions and backgrounds of two professions, drafter and analyst, have converged into the 'modern designer' with a bachelor's degree in either engineering or engineering technology [8]. If this is the case, the course itself should require some divergence from the one engineering faculty participated in as students.

Course Overview and Structure

This paper discusses an undergraduate course in finite element analysis which was only recently adopted for a Bachelor of Science in Engineering (BSE) curriculum. The BSE is a practice-oriented, flexible program which includes a "core" set of required foundational courses in math, science, and engineering, but also allows students flexibility in choosing 30 hours of technical electives. *Introduction to Finite Element Analysis* is one such elective, available to juniors and seniors who have had prerequisite courses in differential equations and solid mechanics. The course format includes two hours of lecture and a two hour computational laboratory. The first half of the semester has focused on theory using one dimensional elements (rods, trusses, and beams). The second half provides practice with two and three dimensional models, incorporating SolidWorks software. Student assessment includes midterm exams, weekly homework exercises on theoretical content (e.g. stiffness matrix calculations), small programming assignments, comprehensive exercises which incorporate physical experiments, and a student-selected semester-long modeling project.

In the sections that follow, this introductory FEA course is discussed with respect to three desired outcomes: (1) establishing a theoretical foundation which builds upon prerequisite courses, (2) providing practical skill with modern design software, and (3) enhancing students' engineering judgment with respect to modeling choices. A contrast of course content and subsequent assessment over two years is also discussed, particularly as it provides insight toward the balance between theory and software.

Building a Theoretical Foundation While Enhancing Students' Understanding of Math and Physics

In general, students take foundational engineering courses in the first two years of an engineering program, the same time frame they take Calculus and Differential Equations. As a consequence, though they may see governing equations in these courses, they quickly become fixated on specific solutions. Revisiting principles of mechanics in the context of FEA reminds students of governing principles. For example, consider the differential equation for an elastic bar as found in Equation 1 where u is displacement in the x direction, P is the internal axial force, and the bar stiffness is dictated by cross-sectional area A and elastic modulus, E:

$$\left(AE\frac{du}{dx}\right) - P(x) = 0 \qquad (1)$$

This form of the governing equation is either missing from typical Mechanics textbooks [9,10] or is presented briefly to derive specific solutions [11] which are literally highlighted for students to use and remember. For example, Beer et al. [10] present three solutions to Equation 1 for a single bar, a series of discrete bar elements, and a continuous bar as given in Equations 2a, 2b, and 2c where δ is the cumulative displacement over length, L.

$$\delta = \frac{PL}{AE} \tag{2a}$$

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i} \tag{2b}$$

$$\delta = \int_0^L \frac{P(x)dx}{AE} \tag{2c}$$

The vast majority of available homework problems in these same texts are discrete. However, a rare continuous example, finding the total deflection of a hanging cable of homogeneous density, may be used to bridge content from Mechanics to FEA. Thus, one of the early course activities is to revisit this problem, applying equilibrium to find load as a function of location and solving for displacement along the entire length using Equation 2c. Students rediscover that as load, P and thus stress and strain vary linearly along the length, the deflection δ varies quadratically. They are then asked to try an alternative approach: segment the bar and lump the total mass at discrete locations. This allows them to approximate the solution using the familiar discrete equation found in Equation 2b. Each discrete segment has constant load, stress, and strain; an obvious divergence from the original continuous problem. In addition, students fill in Table 1 with their predicted displacements and stresses at various locations along the bar. Through this exercise, they realize that their discrete model directly predicts nodal displacement, but they must somehow interpolate between nodes to predict displacement within the element. Thus, this exercise introduces students to the concept of a shape function. Table 1 shows displacement predictions with those obtained through intra-element interpolation (in other words, from shape functions) in a lighter shade. Students also see that while displacement predictions of the elemental models match well at the nodes, stress predictions are far less accurate. Consequently, later decisions on model refinement are dependent on which responses are most important.

Table 1: Predicted Responses for the Hanging Bar of Homogeneous Specific Weight, γ.

Location from Ceiling: x/L	0	1/4	1/2-	1/2+	3/4	1
Model	**Deflection as a fraction of theoretical tip deflection: $\delta / \frac{\gamma L^2}{2E}$**					
Continuous	0	7/16	3/4	3/4	15/16	1
1 Element	0	1/4	1/2	1/2	3/4	1
2 Element	0	3/8	3/4	3/4	7/8	1
Model	**Stress as a fraction of maximum stress: $\sigma / \gamma L$**					
Continuous	1	3/4	1/2	1/2	1/4	0
1 Element	1/2	1/2	1/2	1/2	1/2	1/2
2 Element	3/4	3/4	3/4	1/4	1/4	1/4

Presenting a force (flexibility) method in the opening example helps students connect FEA to earlier coursework. Course lectures and assignments subsequently transition to the direct stiffness method. Students solve spring and bar problems, then use coordinate system transformation matrices to analyze trusses. One dimensional heat transfer rounds out coverage of second order methods. The theory phase of the course finishes with elastic beam analysis. For the second course offering (2014), this included activities to build upon laboratory exercises students had previously conducted in Strengths of Materials and Vibrations classes. These were presented in an earlier paper [12].

The material presented in the first half of the course, more or less, mirrored the theoretical content of a traditional FEA course, relying heavily on the text assigned in 2014 [1]. The rationale was that students needed some theoretical basis to form an understanding of how finite element works and appreciate the care needed to interpret FEA results. For the most part, students completed assignments using Matlab or Excel for matrix algebra. However, student programming skills and familiarity with Matlab varied greatly. This presented challenges and slowed the anticipated pace of the course. As an example, one lecture posed a typical truss problem requiring the following general steps: write the elemental stiffness equations, perform coordinate transformation, assemble the global matrix, solve the global stiffness equation, and finally determine elemental stresses. In the first offering (2013), a five bar truss example was demonstrated in Excel, generally following an outline offered in the textbook [6]. Though matrix algebra is very cumbersome in this platform, there were some decided advantages to using Excel. For one, students have a relatively high comfort level with the program. In addition, color coding along with 'blank' empty cells made the global matrix assembly process more transparent. Even so, completing the example took much longer than necessary to convey the general process. Instead, considerable effort went into troubleshooting minor mistakes as would, in hindsight, seem inevitable when an entire class is trying to assemble five or more element matrices. In 2014, the instructors decided to forgo Excel in favor of Matlab to solve a similar example. However, little improvement was made in the efficiency of the exercise. Students later attempted to use Matlab to solve assignments on their own. Most were successful, but a few struggled to see the big picture from beyond a host of syntax issues.

The second half of the course was to focus more on using commercially available software, particularly to solve problems with two and three dimensional elements. This aspect is discussed in the following section.

Developing Technical Competency

Students bring some proficiency in the use of solid modeling software from their freshman design sequence, internships and co-op work, and even high school. As they encounter complex design projects in upper-level courses, they are naturally empowered to employ this skill. Seeing what they assume will be a seamless transition from solid model to discretized FEA model, they then look to the same software to help them predict behavior (e.g, determining elastic deformations and stresses). It seems both powerful and easy. However, the potential for inexperienced users to make a host of modeling errors is well documented [8,13–15]. These include, among other things: improperly defining boundary conditions, selecting the wrong types of elements, not understanding the underlying assumptions of the analysis, false confidence in unverified results, etc. Realizing it would be impossible to build proficiency in a single semester course, the authors aimed to provide introductory level experience which would foster an appreciation of the power of FEA, supported by enough theory to give them a healthy appreciation of the care required to interpret results.

Seeking to integrate theory and software, the 2013 class was assigned an SDC Publications textbook by Randy Shih, based on SolidWorks Simulation [6]. The text provides some background on the Direct Stiffness method and progresses from trusses to beams, to 2D elements, 3D solid elements, and finally thin shell analysis. Each chapter has case studies which include illustrated step-by-step directions with practical details such as how to set up ‘weldment’ geometries for 1D elements, how to apply various boundary conditions including symmetric constraints, how to refine the mesh and check for convergence, as well as how to obtain and represent results. It seemed to align well with the goals and framework for the course.

Using SolidWorks to enforce basic FEA concepts was more difficult than expected. For example, from a theoretical standpoint, it seemed logical to start with one dimensional bar elements and add complexity in the order outlined earlier (e.g. truss, beam, 2D, 3D). However, solid element analysis is much more straight-forward than 1-D analysis in SolidWorks. As a case in point, DassaultSystèmes’ offers a “Simulation Student Guide" which walks users through stress analysis of a spider hub assembly using solid elements [16]. There are seven basic steps: (1) create the study, (2) assign material, (3) apply fixtures, (4) apply loads, (5) mesh, (6) analyze, and (7) view results. Most users could complete the exercise in well under an hour. Conversely, Shih (in the SDC text) offers students two ways to create a truss FEA model, neither of which is nearly as intuitive. The first approach is to draw solid bars to provide a representation of the ‘true’ geometry without a priori consideration for analysis. For the truss, this involves creating special planes from which cross-sectional geometries can be extruded. Then, an FEA model is obtained by ‘‘treat[ing] the selected bodies as beams," editing the beams to make them ‘truss’ elements, and calculating joints to form nodes. So *truss* elements are treated as a subset of *beams*, and users must override two default choices in order to get the simplest mesh. Furthermore, if users attempt to start with a solid model which has more detail in the connections, establishing nodes can be more challenging. Alternatively, one-dimensional structural members are by default *beam* elements. These are created using a ‘weldment’ profile which must be predefined in a particular installation directory. This presented a minor issue for students working on university computers for which they were not authorized access to the hard drive. A more frustrating finding was a glitch in the results table for member stresses. Member forces were correctly obtained, and the stress legend in the main display window was correct. However, the *List Forces display* of stresses (force/area) is incorrect by several orders of magnitude. In short, SolidWorks Simulation was found to be clunky overkill for truss analysis.

Another frustrating point with SolidWorks involved the translation of boundary conditions in SolidWorks terms. Options for constraints for *truss* joints include ‘fixed,’ ‘immovable,’ and ‘reference geometry.’ Thus reference geometry is needed to specifically control the three translation and three rotational degrees of freedom. All directions are relative to this geometry and indicated with figures representing the perpendicular direction, and two orthogonal directions. One can specify standard x, y, z directions using the three initial planes, ‘front,’ ‘top,’ and ‘right,’ but this can get confusing for new users. Solid elements have several additional support options including ‘fixed hinge,’ ‘bearing fixture,’ a host of symmetry constraints among others. Again, the translation between these and the six degrees of freedom is not always obvious. Establishing force boundary conditions appears limiting as well. In general, forces require an edge or surface from which to establish location and direction, and this must be done prior to meshing. They can be applied to joints in a beam or truss model, but not nodes for solid and shell elements.

Despite its shortcomings, the truss example in SolidWorks did have some pedagogical benefit. Using an exercise from Kurowsksi’s text, students were asked to duplicate a study with truss elements and then beam elements[17]. Beam elements have three nodal degrees of freedom (2D displacement and rotation) whereas truss elements only have one (axial displacement). In addition, beam connections are modeled as rigid in SolidWorks while truss elements are pin-connections. From the exaggerated displacement, shown in Figures 1 and 2 students observe the consequences of this modeling choice.

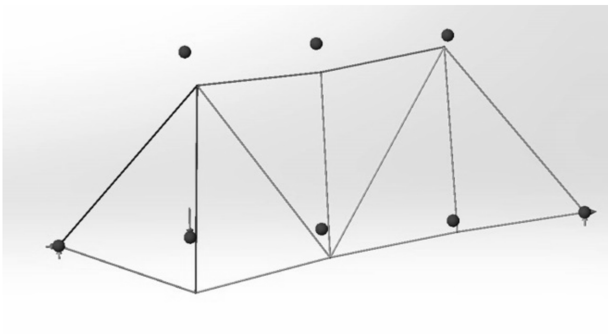

Figure 1: SolidWorks Simulation Screenshot from Truss Analysis based on Kurowski.[17]

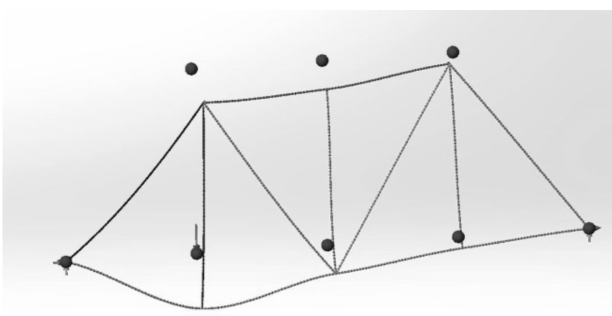

Figure 2: SolidWorks Simulation Screenshot from Beam Analysis based on Kurowski.[17]

In a similar fashion to the truss/beam example, the 2013 class followed a case study for a C-clamp from their text which contrasts results from a 2D and 3D model [6]. In this same example, they learned to refine their mesh and look for convergence of maximum stress. The SolidWorks exercises provided practical experience needed for the student-selected semester project.

Experiencing the Big Picture: Modeling Choices and Engineering Judgment

In addition to showing students the connections between earlier courses and developing a useful design skill set, the FEA course provides insight to the nature of modeling and analysis. As statistician George Box famously quipped, “Essentially, all models are wrong, but some are useful [18]." This is an unsettling concept for undergraduates, who are most often encouraged to seek a single ‘right’ answer (i.e. the one found in the back of textbooks). However, students of the finite element method have no choice but to consider the impacts of their modeling choices including boundary conditions, element type, mesh size, etc. A semester project was assigned to get students to consider these effects.

The project required students to develop a set of models for an engineered system of their choice. One model had to be a coarse, analytical model. A minimum of three additional models were to be constructed with FEA software, each with some reasonable distinction in how they approximate the same system, e.g. what elements they used, how boundary conditions were applied, etc. They were to discuss differences in their results and provide a reasoned judgment as to which results were ‘trustworthy.’ Instructors evaluated three written deliverables: a proposal, preliminary and final report. On the final day of class, students gave oral presentations of their analyses to one another. Project examples included an archery bow, a glider airplane wing, a Jet Ski dolly, a Swiss Army knife, and a truck tailgate being stamped in the factory. Students primarily elected to study the effects of boundary conditions (particularly how forces were applied) and geometry factors. In the first offering, a few students failed to grasp the intent of the assignment, focusing more on actual design decisions rather than modeling choices. This was somewhat better during the second offering in 2014. However, a number of common mistakes persisted:

- Failing to assess which models were most faithful to the real system.
- Assuming the analytical model is the 'right' answer.
- Making trivial changes.
- Failing to show mesh convergence.
- Failing to consider the effect of stress concentrations.
- Assuming that material strength will affect elastic response.

At the same time, the projects revealed growth in understanding of modeling in general and the finite element method in particular. Students were able to clearly articulate the responses of interest for their designs. Most showed that their results converged after mesh refinement. Many demonstrated appreciation for differences in their analytical and FEA models, e.g. a student studying bicycle training wheels saw the significance of assumptions about connections. Several students made thoughtful choices about loading boundary conditions. e.g. a student studying a rolling chair looked at how to alter boundary conditions based on how the user was sitting.

A few changes in the project requirements for future offerings should help improve the final product. The 2014 class seemed to gravitate toward systems they could model with beam elements. This was likely influenced by the fact they had fewer SolidWorks exercises than their predecessors. This isn't necessarily a negative, but some students selected overly simple systems. Establishing a more specific grading rubric that includes degree of difficulty in addition to the quality of the analysis should help significantly.

Course Assessment from 2013 to 2014

In 2013, the chosen textbook [6] focused on SolidWorks applications, which the instructors supplemented with early theoretical content during lecture. Students showed excellent grasp of practical concepts, e.g. distinguishing key differences between beams and truss elements, understanding the steps of the FEA process, etc. This was demonstrated by a 97% average on high-level conceptual exam questions. On the other hand, students showed less mastery of more fundamental theoretical ideas. For example, the course average was 78% for a problem requiring they construct a global stiffness matrix for beam elements and only 63% for one requiring they use beam shape functions to predict deflections in-between nodes. In the end, student feedback for the course was overwhelmingly positive. One hundred percent 'strongly agreed' they learned a lot, and that assignments helped them do so. They had mixed reviews of the text, and some wanted even more SolidWorks exercises. In particular, one student mentioned needing the SolidWorks activities earlier in the semester to help with the project.

In 2014, student perceptions of the course fell significantly. Though 72% still at least 'agreed' that they learned from course assignments and lectures, fewer 'strongly agreed', and a few students were uncertain about the value of the course. A few critical comments included wanting more SolidWorks and overly long lectures. In fact, the `balance' struck between theory and software fell more heavily on the theoretical side for this offering. A more traditional, introductory textbook [1] was selected. The SolidWorks truss example was omitted, and the assigned programming projects were slightly more substantial. Exam metrics reflected this change in emphasis as well. Students were more capable of generating global beam stiffness matrices by hand (87%), and slightly more familiar with shape functions (70%). However, nearly half of students could not answer a conceptual question regarding the difference between a finite element and continuous solution for an elastic bar.

It should be noted that both class sizes were small (11 and 15), and that there were differences in expectations in each group. In 2013, the students were nearer completion of the degree, with more experience from upper level courses with a significant programming component. Specifically, 45% of the 2013 cohort had taken two or more such courses and 36% had taken at least three. Of the 2014 class, only 33% had programming experience from more than one technical elective. Furthermore, it appeared that several students in the 2014 cohort made more direct connections between solid modeling and FEA (many from experience with internships), and were harder to convince of the value in theory. Thus, more practical experience with software is needed to keep students interested, but work also needs to be done to communicate expectations of the course content.

Conclusions

The authors sought to design a course suitable for an undergraduate BSE program which has an appropriate amount of theory while providing practical experience with FEA software.

The paper has discussed several challenges, particularly relating to finding a balance between theory, use of software, and developing engineering judgment. Finite element analysis theory provides an opportunity to cement ideas from earlier coursework. However, heavy programming assignments may not be the best way to communicate these concepts with this demographic of student. SolidWorks Simulation software has significant potential to help convey concepts as well as to provide experience with the practical application of the finite element method. For one, students already have comfort with SolidWorks as a solid modeling tool from freshman design courses. Another advantage is that there is an abundance of resources with case studies that demonstrate concepts. However, translating some FEA principles, especially for one-dimensional elements can be confusing. Finally, engineering judgment is arguably the most important asset a student can gain from any academic course. An FEA course has the opportunity to affect this kind of development by requiring students to combine learning from analytical models, physical experiments, and software based on numerical techniques.

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Biographical Information

Natasha Smith is an Associate Professor of Engineering at the University of Southern Indiana. She teaches courses in mechanics, materials, thermodynamics, and machine component design in addition to FEA. She earned a B.S. in Ocean Engineering from the U.S. Naval Academy as well as an M.S. and Ph.D. in Civil Engineering from Vanderbilt University. After a combined active and reserve career in the U.S. Navy, she joined the faculty at USI in 2009. Her research interests include reliability-based design, multidisciplinary optimization, and materials testing methods.

Julian L. Davis is an Assistant Professor of Engineering with the University of Southern Indiana (USI). He received his B.S. and Ph.D. in Engineering Mechanics from Virginia Tech. At USI he teaches freshman design courses, statics, dynamics, vibrations & FEA. His research interests include the application of finite element modeling to the study of biological systems. His educational research includes the investigation of technical communication through online homework systems and best practices for teaching FEA.