Parametric Engineering Design: Integrating Analytical Methods with CAD and Simulation

Solomon Gilbert Diamond
Thayer School of Engineering at Dartmouth, Hanover, NH, USA

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Introduction
In mechanical engineering design instruction, there is an artificial separation between the analytical methods that are taught in a traditional mechanical engineering course and computer-aided design (CAD) software tools. Engineering undergraduates in the “digital generation” often have the opinion that analytical engineering approximations are increasingly irrelevant in the modern digital world. To them modern CAD and finite element mesh (FEM) simulation appear to offer more power and accuracy for less work. However, skipping the analytical methods leads to a loss of fundamental insight, with a guess-and-check approach to design substituting for closed-form solutions. The solution to this challenge in education is to integrate analytical methods directly into the CAD and simulation environment and to demonstrate the power and relevance of Mechanics in a modern design context. This work presents sample problems from an undergraduate course in Solid Mechanics at the Thayer School of Engineering in which design problems are solved analytically and then combined CAD and simulation.

It is helpful to the ensuing discussion to draw a distinction between forward and inverse calculations in mechanics. In forward calculations, one is given information such as linear dimensions, material properties, restraints, and applied loads. Analysis is then performed to derive quantities such as deformed geometry, stress, strain, and reaction forces. In a mechanical design, one or more of the derived quantities is specified and an inverse problem is solved, for example, to optimize a part dimension or calculate a material property requirement. The value of analytical engineering is particularly clear in design optimization problems where solutions can be functionally expressed in terms of the design constraints. CAD models and simulation can analyze specific instances of complex geometry, loads, and constraints but offer no comparable general solution.

In the following parametric engineering design examples, analytical solutions are coded directly into CAD models to drive the primary model geometry. Simulation is then used to refine detailed aspects of the design. Incorporating the analytical solutions into CAD leads to closed-form parametric models that automatically update when design specifications are changed. The CAD and simulation-only alternative leads to non-parametric design models that require an iterative search to re-optimize the design. The integrated approach is a best practice in real-world design that also presents an exciting opportunity in engineering education. Analytically-driven CAD models turn mathematical expressions into virtual parts and FEM simulation allows students to check and visualize solutions with an independent methodology.

From a pedagogical standpoint, there are four steps to parametric engineering design. These steps are patterned after the problem solving steps in solid mechanics [1]. First is to specify design intent. This step entails listing design constraints, assigning nomenclature, and
drawing freehand sketches as needed to describe the problem. The parameters are divided into those key quantities that will drive design and those that will be computed or optimized. The second step is to solve the problem analytically. Third is to build the parametric model in CAD from the analytical solution. Fourth is to confirm and adjust model with simulation. These steps are illustrated in the following design problems for a simple beam and planar truss. The CAD models were all generated using SolidWorks® 2009, Student Edition (Dassault Systèmes SolidWorks Corp., Concord, MA, USA) on a laptop PC with a dual core 2.26 GHz processor and 2 GB of RAM.

Example 1: Parametric Design of a Beam

The uniformly loaded simple beam in Fig. 1 is completely fixed at its two ends. The material is assumed to be 6063-T6 aluminum. The beam has a rectangular cross section as shown with cross-sectional moment of inertia $I = bh^3/12$. The problem statement is that for a given load $F$, beam length $L$, and beam width $b$, find the beam height $h$ that achieves a vertical deflection of less than 0.1% of $L$. For a load in the negative $y$-direction, the design intent is captured by the equation

$$v_{\text{max}} = -0.001L.$$  \hspace{1cm} (1)

One approach to solving this problem analytically is to apply the forth order integration method to solve the load-deflection equation

$$EI \frac{d^4v}{dx^4} = P$$  \hspace{1cm} (2)

subject to the boundary conditions

$$v(0) = 0, \quad v(L) = 0, \quad \frac{dv(0)}{dx} = 0, \quad \frac{dv(L)}{dx} = 0,$$  \hspace{1cm} (3)

and obtain the forward solution for vertical displacement $v$ as a function of $x$

$$v(x) = \frac{PL^4}{24EI} \left[ \left( \frac{x}{L} \right)^4 - 2 \left( \frac{x}{L} \right)^3 + \left( \frac{x}{L} \right)^2 \right].$$  \hspace{1cm} (4)

The inverse solution is then obtained by combining the design intent (1) with the geometry of deformation (4) at its maximum value $v_{\text{max}} = v(L/2)$, and solving for $h$

$$h = 5L \left( \frac{-P}{4Eb} \right)^{\frac{1}{3}}.$$  \hspace{1cm} (5)
Considering the specific case of $L = 20$ in, $F = -1000$ lb, $P = F/L = -50$ lb/in, $b = 0.5$ in, and $E = 10.0076 \times 10^6$ psi (6063-T6), the solution is

$$h = 5L \left( \frac{-P}{4Eb} \right)^{\frac{1}{3}} = 1.36 \text{ in}$$

(6)

It is also relevant to check the maximum stress due to bending, which occurs symmetrically at $x = 0$ and $x = L$. This calculation begins by evaluating the moment-curvature equation at $x = 0$

$$M = EI \frac{d^2v}{dx^2}, \quad M(0) = \frac{-PL^2}{12} = 1.67 \times 10^3 \text{ in-lb}.$$ 

(7)

Next the flexure formula is evaluated at $y = h/2$, the top surface of the beam, to determine the axial stress

$$\sigma_x = \frac{-My}{I}, \quad \sigma_{x,\text{max}} = \frac{-PL^2}{2bh^2} = 1.09 \times 10^4 \text{ psi}, \quad \sigma_{\text{yield}} = 3.12 \times 10^4 \text{ psi}.$$ 

(8)

A parametric CAD model is then created from the analytical solution (Fig. 2). Although the CAD model is geometrically nothing more than an extruded rectangle, the analytical equations are directly linked to the geometry. This means that if the design force $F$ is altered, the beam height will automatically adjust to accommodate the change.

![Fig. 2. Parametric model of a beam (top) and the parametric design equations (bottom) that were entered into the CAD software.](image)

The parametric design behavior of the beam model is fundamentally different from the typical CAD approach where the beam height $h$ would be initially guessed and then optimized by a search algorithm using FEM simulations at every step. This sort of brute force solution to the beam design problem (Fig. 3) must be restarted every time a design parameter changes. The computational load of a parameter search using FEM simulation can be considerable even for trivial geometries with a single unknown parameter like in this beam example, which takes about 30 seconds to solve on the reasonably fast PC described in the Introduction. More complex geometries with more than a few unknown parameters can make the search computationally intractable. Given the high computational cost of FEM simulation, it is best applied when analytical methods are less tractable.
Fig. 3. Global maximum displacement (left) shows that for the design deflection of 0.02 in, the optimal beam height is $h = 1.38$ in. Global maximum von Mises stress (right) shows that the corresponding maximum stress is $1.12 \times 10^4$ psi.

The correspondence of analytical and simulation solutions is quite good. The beam height results between the analytical and simulation solutions differ by 1.4%. The maximum stress results differ by 3.5%. The CAD environment also permits us to compare the geometry of deformation. For this the simulation result showing the deformed beam is exported into a part file. Then a new part is created using the analytical geometry of deformation (4) as a sweep path for the beam cross section. The two parts are then overlaid and compared (Fig. 4). In this case there is no need for refinement of the analytical solution.

Example 2: Parametric Design of a Truss
A planar truss is shown in Fig. 5 having three joints and three links. The problem definition gives the material as plain carbon steel. Links 1 and 3 have round x-sections, and link 2 is a pair of bars with identical rectangular x-sections. The locations of joints A and C relative to B are subject to change and should be parameterized. Other key parameters are the load $P$, and the factor of safety (FOS) with respect to tensile failure in links 1 and 2, shear failure in joint pins A, B, C, and buckling in link 2. The design intent is to minimize material usage in the links and pins subject to the design constraints and FOS requirements.

The first step in the analytical solution is to solve for the internal axial forces in the links. As a statics problem, this can be accomplished by force equilibrium at joint C (Fig. 6). The sum of forces in $x$-direction is zero

$$-F_1 \cos(\theta_A) - F_2 \cos(\theta_A) = 0, \quad (9)$$

and the sum of forces in $y$-direction is zero
\[ -F_1 \sin(\theta_A) - F_2 \sin(\theta_B) - P = 0. \]  

The resulting internal forces are

\[ F_1 = \frac{P}{\sin(\theta_A) + \cos(\theta_A) \tan(\theta_B)} , \quad F_2 = \frac{-F_1 \cos(\theta_A)}{\cos(\theta_B)}. \]  

The axial stress in links 1, 2, and 3 are then obtained using parameters for the cross-sectional areas of the links

\[ \sigma_1 = \frac{F_1}{A_1}, \quad \sigma_2 = \frac{F_2}{A_2}, \quad \sigma_3 = \frac{F_3}{A_3}. \]

A factor of safety (FOS) analysis is now performed on link 1 with respect to its tensile load

\[ FOS = \frac{F_{\text{failure}}}{F_{\text{allow}}}, \quad F_{\text{allow}} = F_1 \]

\[ A_1 = \frac{F_{\text{failure}}}{\sigma_{\text{yield}}}, \quad A_1 = \frac{\pi d^2}{4} \]

to obtain an expression for the optimal cross-sectional diameter \( d_1 \) of link 1

\[ d_1 = \sqrt{\frac{4 F_1 FOS}{\sigma_{\text{yield}} \pi}}. \]

A simple shear analysis is next performed on the joint pin A (Fig. 7) using an empirical relation between the yield stress in shear and the yield stress in tension \( \tau_{\text{yield}} \approx 0.577 \sigma_{\text{yield}} \). The FOS analysis
\[ V_A = \frac{F_1}{2}, \quad FOS = \frac{\tau_{\text{yield}}}{\tau_A} , \]
\[ \tau_A = \frac{V_A}{A_A}, \quad A_A = \frac{\pi d_A^2}{4} \]

results in an expression for the optimal pin diameter \( d_A \)
\[ d_A = \sqrt{\frac{2F_1 FOS}{\tau_{\text{yield}} \pi}} . \] (16)

Lastly, a buckling analysis is performed on link 2 (Fig. 8) to obtain an expression for the optimal cross-sectional link height \( h_2 \) given a fixed link base \( b_2 \) and critical force for Euler buckling

\[ F_{\text{critical}} = \frac{\pi^2 EI_2}{L_2^2}, \quad I_2 = \frac{b_2 h_2^3}{12} \]
\[ \frac{F_2}{2} = \frac{F_{\text{critical}}}{FOS}, \quad h_2 = \left( \frac{-6F_2 I_2^3 FOS}{\pi Eb_2} \right)^{\frac{1}{3}} . \] (17)

A parametric truss assembly is then constructed from the analytical solutions (Fig. 9). The assembly contains a layout sketch similar to Fig. 5 and separate parts for each of the links and pins with geometries driven by the analytical equations. The location of joint C may be changed in the layout sketch and this change will automatically propagate through all the parts. Changes to the applied load or FOS will also automatically result in a redesign of all the pin diameters and link cross sections. Details of the CAD model such as the fillet radii and yolk width are linked to appropriate parametrically driven geometries so that they also scale with changes in the overall geometry, loading conditions, and FOS.
During the initial development of the parametric truss model, FEM simulation was used to fine-tune details of the model. For example, several stress concentrations were found to occur at the butt joints of links. These stress concentrations were ameliorated by increasing the fillet radii and re-checked with additional simulations. Simulation was also used as a comparative FOS analysis on the links and pins. The FOS analysis on links 1 and 3 agrees closely between the

\[ \text{"FOS"} = 3 \]
\[ \text{"E"} = 30.4579 \ \text{lb x 10^6 in} \]
\[ \text{"Yield Stress"} = 31.9944 \ \text{lb x 10^3 / in}^2 \]
\[ \text{"Allowed Stress"} = \frac{\text{"Yield Stress"}}{\text{"FOS"}} = \text{"lb x 10^3 / in}^2 \]
\[ \text{"Max Load P"} = 10 \ \text{lb x 10^3} \]
\[ \text{"F1"} = \frac{\text{"Max Load P"}}{\sin(\text{"thetaA"}) + \cos(\text{"thetaA"}) \cdot \tan(\text{"thetaB")}} = \text{"lb x 10^3} \]
\[ \text{"F2"} = \frac{\text{"F1"} \cdot \cos(\text{"thetaA"})}{\cos(\text{"thetaB")}} = \text{"lb x 10^3} \]
\[ \text{"Area1"} = \frac{\text{"F1"}}{\text{"Allowed Stress"}} = \text{"in}^2 \]
\[ \text{"Area2"} = \frac{\text{"F2"}}{\text{"Allowed Stress"}} = \text{"in}^2 \]
\[ \text{"Area3"} = \frac{\text{"Max Load P"}}{\text{"Allowed Stress"}} = \text{"in}^2 \]
\[ \text{"Dia1"} = \sqrt{\frac{4 \cdot \text{"Area1"}}{\pi}} = \text{"in} \]
\[ \text{"Dia3"} = \sqrt{\frac{4 \cdot \text{"Area3"}}{\pi}} = \text{"in} \]
\[ \text{"h2 buckling"} = -\frac{6 \cdot 12 \cdot 2 \cdot \text{"FOS"} \cdot \frac{\text{"F2"}}{\pi}}{10^3 \cdot \text{"b2"}} \cdot (1/3) = \text{"in} \]
\[ \text{"h2 yield"} = \frac{\text{"Area2"}}{\text{"b2"}} \cdot 2 = \text{"in} \]
\[ \text{"h2"} = \text{if}(\text{"h2 buckling"} > \text{"h2 yield"}, \text{"h2 buckling"}, \text{"h2 yield"}) = \text{"in} \]
\[ \text{"Pin A dia"} = \sqrt{\frac{2 \cdot \text{abs("F1")}}{\text{"Allowed Stress"} \cdot \pi}} = \text{"in} \]
\[ \text{"Pin B dia"} = \sqrt{\frac{2 \cdot \text{abs("F2")}}{\text{"Allowed Stress"} \cdot \pi}} = \text{"in} \]
\[ \text{"Pin C dia"} = \sqrt{\frac{2 \cdot \text{abs("Max Load P")}}{\text{"Allowed Stress"} \cdot \pi}} = \text{"in} \]

Fig. 9. Parametric model for the truss problem (right) and the analytical equations as entered into the CAD software (right).

Fig. 10. Factor of safety analysis results from FEM simulation showing that the design meets the FOS = 3 specification in the links in a static simulation (left). Buckling mode simulation (right).
analytical and simulation solutions (Fig. 10). The buckling analysis yielded an effective FOS of 3.5, which is 16.7% greater than the analytical design FOS of 3. This discrepancy can be explained by the assumption in Euler buckling that pin joints support the links when in this design, link 3 is additionally constrained by the structure of joints B and C. These additional constraints effectively shorten the length of link 3 and increase the critical load for buckling. Reasoning through the discrepancies between analytical and simulation solutions is one source of developing design insight.

**Discussion**

Combining analytical and simulation approaches in a single integrated parametric model offers advantages to the practice and instruction of mechanical engineering design. In practice, the parametric approach leads to more efficient design optimization because computational power is directed to the aspects of design problems where simulation is most needed. In education, the integrated approach transforms analytical solutions from equations on paper to dramatically illustrated solid models. Students can also use simulation to cross check analytical results. When used in the weekly homework assignments of a Solid Mechanics course at the Thayer School of Engineering, students spontaneously questioned and discussed solution discrepancies. They were motivated to re-examine their analytical methods and to question assumptions in simulation. The CAD environment and simulation is in effect a digital laboratory for students to test solutions with numerical experiments. The students also appeared to develop greater mechanical intuition as evidenced by the overall class performance in a “bridge crushing contest.”

**References**


**Author Biography**

Solomon Gilbert Diamond was born in Bath, New Hampshire. He received his AB in engineering sciences from Dartmouth College, Hanover, NH, in 1997 and then a BE from the Thayer School of Engineering at Dartmouth in 1998. He received a PhD in engineering sciences from Harvard University, Cambridge, MA in 2004, and completed post-doctoral training at the Martinos Center for Biomedical Imaging, Massachusetts General Hospital, Charlestown, MA in 2007. He is currently an Assistant Professor at the Thayer School of Engineering at Dartmouth. His research applies noninvasive functional neuroimaging technologies to study human brain physiology. He uses a multimodal instrumentation approach to monitor neural activity and cerebrovascular hemodynamics with the goal of improving the early diagnosis and monitoring of neurodegeneration. Prof. Diamond is a member of the IEEE, the ASEE and Tau Beta Pi.

Solomon G. Diamond, PhD, Assistant Professor
Thayer School of Engineering at Dartmouth
8000 Cummings Hall
Hanover, NH 03755
Solomon.G.Diamond@Dartmouth.edu