The Use of Classroom Case Studies in Preparing First-Year Mathematics Graduate Teaching Assistants

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Introduction

In 2000, 7.2% of all students enrolled in regular stream Calculus I in the United States were taught by mathematics graduate students who served as instructors of record and had sole or primary responsibility for all course activities. By 2005, that number had grown to 10.7%. The 2010 data collection was incomplete: 8% of Calculus I students were with instructors whose classification cannot be determined, but at least 10% (and almost certainly more) were taught by graduate students. In departments offering an M.S. or Ph.D. in mathematics, these rates were much higher: 10.4% in 2000, 13.9% in 2005, and at least 15.4% (with 12.2% undetermined) in 2010. Among Ph.D. granting institutions, the number was over 20% in both the 2005 and 2010 studies.

Given that 54% of all undergraduates attend institutions offering at least a Master’s degree, this implies that a significant portion of STEM majors in their first year of college mathematics are taught by graduate teaching assistants (GTAs). Since performance in STEM-related courses is a key factor in attrition rates for STEM majors, the teaching preparation of mathematics graduate students is a critical issue for improving pathways to success for undergraduate STEM majors.

There have been many laudable programs aimed at improving the professional preparation of graduate students and new faculty members, particularly in the STEM disciplines. The Preparing Future Faculty Initiative by the Council of Graduate Schools (CGS) funded multiple programs with this goal. One outcome of those pilot programs was a specific call for changes in the preparation of future mathematics and science faculty. More recently, the CGS initiative took on the narrower focus of preparing future faculty across multiple disciplines to assess student learning. Among the changes urged by these two reports is a call to change the academic culture so that the false “teaching vs. research” dichotomy is shifted to a paradigm of “teaching as research/scholarship.” In this context, institutions are charged with helping graduate students develop more effective teaching strategies and a better understanding of students as learners.

Professional programs for secondary mathematics teachers have long held the goals now proposed for graduate student preparation. Indeed, secondary education certification in most states requires significant coursework in learning theory and pedagogy. It is not practical to require GTAs to complete the same amount of education coursework along with their subject matter coursework,
nor is it appropriate: the student population for secondary mathematics and for university teaching are quite different, as are the contextual issues associated with higher education. However, it is worth examining effective practice in the preparation of secondary teachers to see what components might be translated appropriately to graduate student preparation.

The use of case study in professional preparation has a long history, not only in law, business, medicine, and engineering, but more recently in K – 12 teacher preparation. It has also been suggested as an effective method for preparation of graduate students. It is the use of case study in the professional preparation of mathematics GTAs that we examine in the current paper.

We embedded four first-year GTAs in a senior-level course for secondary mathematics education majors and used classroom mathematics case studies as a central component of the course. In this context, we seek to answer these questions:

1. To what extent, and in what manner, did the nature of the graduate students’ comments during case analysis change over time?
2. To what extent, and in what manner, did the graduate students’ perceptions of teaching and of themselves as teachers shift over time?
3. How did the graduate participants perform as first-time teachers of record, compared to first-time teachers of record who were not embedded in the secondary education course?

Two of our research questions are qualitative in nature, and our chosen research methods reflect that. Rather than conduct a quasi-experimental design with a selection of GTAs participating in case analysis and others not, we instead used mixed qualitative and quantitative methods to collect and analyze data solely from participants who experienced the use of case analysis in their first semester of graduate school. This paper focuses in particular on two quantitative measures (survey data and student performance) and on two qualitative measures (case discussion records and reflective writings). We give a summary of the data within each of those four categories separately. However, the nature of the research questions is such that a more significant analysis involves integration of those data to gain a clearer picture of the experience of each individual participant. Then we can look for similarities among, and differences between, the study participants to seek insight into the research questions.

Research Design

Institutional and Participant Background

We placed four first-year mathematics graduate students within a senior-level mathematics course for secondary education majors. For simplicity, we will refer to that course as the “combined course.” The combined course is required in the fall semester for all senior secondary mathematics education majors and only one section is offered each year. There were 18 undergraduates enrolled during the semester that graduate students were embedded in the course. The course is intended to help students develop a deeper understanding of the mathematical content and effective pedagogy for the secondary mathematics curriculum. All of the graduate students were assigned as teaching assistants in a precalculus course covering the same content as
that in the secondary mathematics curriculum. Four of the undergraduates also assisted in the same precalculus course. Thus, the field of interactions among the students was as shown in Figure 1.

Figure 1: Areas of interaction for the undergraduate and graduate students in the study. “U” indicates an undergraduate student, “G” indicates a graduate teaching assistant and “xG” indicates a graduate student whose data was not used in the study.

Graduate participants in the study were selected solely on the basis of availability during the class times for precalculus and for the combined course. Five graduate students were available for both. Of those, four were full-time GTAs in their first year of graduate school. The fifth was a third-year graduate student on part-time assistantship who had previous experience as a teacher of record. The four first-year GTAs form the subjects of this study. The third-year GTA participated in the combined course but is not included in the study.

Of the four subjects, two were men and two were women. All were native English speakers and U.S. citizens who had received undergraduate degrees in mathematics from private four-year colleges in the United States. All four had experience as mathematics tutors during their undergraduate studies, but none had been teacher of record for any course at any level. One had pursued a K-12 teaching certificate but had chosen to attend graduate school rather than complete the student teaching requirement to obtain a full credential. One of the men and one of the women completed summer coursework to remedy undergraduate academic deficiencies in statistics and abstract algebra prior to beginning graduate school. Their undergraduate cumulative grade point averages (CGPA) were 3.52, 3.52, 3.95, and 4.0, as compared to a mean undergraduate CGPA of 3.81 among all incoming mathematics graduate students that year.

Because the precalculus course was central to the structure of the combined course, we briefly describe its structure as well. The precalculus course is taken by STEM majors whose placement scores prohibit them from enrolling directly in Calculus I. The content covers essentially all of high school mathematics, covering topics from adding rational numbers through trigonometry and conic sections. The course structure is hybrid, with students working independently outside of class using an online learning and assessment program called ALEKS® (Assessment and LEarning in Knowledge Spaces). Each face-to-face meeting has 60-70 students in two adjoining classrooms with one faculty member, two to three GTAs, and zero or one undergraduate assistant. Reporting features in ALEKS® allow for targeted individual instruction during the twice-weekly 75-minute face-to-face meetings. Students receive direct instruction only on topics they have attempted but have been unable to master on their own using the online instructional materials. Direct instruction is given to small groups where appropriate, and to individuals when warranted.
Most direct instruction periods last 10 - 20 minutes of the 75-minute period. Precalculus students progress at their own pace and must complete all learning objectives in order to pass the course. There are no scheduled exams covering fixed content; online assessment is ongoing.

Throughout the semester, students in the combined course worked in teams consisting of one GTA, one undergraduate precalculus assistant, and two or three other undergraduates. There were five different team assignments over the course of the semester, ensuring that each GTA worked with each undergraduate precalculus assistant and with all or nearly all of the other undergraduates. The content of the combined course was closely connected to the precalculus classrooms at the university and to cooperating teacher classrooms at the high schools. Pedagogical content knowledge was addressed directly and repeatedly, as were reflection on practice and professional identity.

**Use of Cases in the Combined Course**

In the 1990’s, the Harvard Mathematics Case Development Project (HMCDP) sought to establish a basis of cases for the preparation of mathematics teaching professionals. Several of those cases were published as *Windows on Teaching Math: Case Studies in Middle and Secondary Classrooms*[^22], which we used as the sole required text for the combined course. Each of the published cases includes pre-case prompts, the case itself, and post-case prompts. Of the eleven cases available in the text, the six most closely linked to the content of the precalculus course were used for discussion in the combined course, as indicated in Table 1. One additional draft version of a case from the HMCDP was used as well; it is indicated in the table as unpublished.

Each of the seven case arcs consisted of a pre-case activity in class, individual reading of the case out of class, and in-class guided post-case discussion. The pre-case activities were primarily mathematical in that they were designed to have the combined course participants carry out the mathematical tasks of the case. In some instances, the pre-case activities also included pedagogical prompts. In general, the pre-case activities were conducted individually in class as an exit activity. The post-case discussion prompts fell into four broad categories: mathematical issues, analyzing student thinking, pedagogical issues, and contextual issues. Every case discussion included discussion of the mathematical issues and either student thinking or pedagogical issues (or both). Some post-case discussions included prompts from all four categories. For each post-case discussion, teams assigned a recorder for each prompt and recorded the contributions of each individual team member. Thus, we are able to look at the individual contributions made by each GTA in the discussion of each prompt. Although seven cases were used in the course, the mathematics for one case, Ships in the Fog, proved to be sufficiently challenging that the case discussion never reached pedagogical or contextual issues, but centered solely on developing the necessary mathematical skills.

In order to better understand the manner in which cases were used, and thus the nature of the data derived from case artifacts, we provide here fuller detail for a single case arc: Slippery Cylinders. This was the second case and it was covered in the third week of the semester when the bulk of the precalculus students were just starting to work on a course objective related to connections between geometry and algebra.

The pre-case activity was administered during the last twenty minutes of class. Groups were given two sheets of identical blank paper and instructed to create both a tall cylinder and a short
<table>
<thead>
<tr>
<th>Case Title</th>
<th>Mathematical Content</th>
<th>Pedagogical &amp; Contextual Issues</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lost in Translation</td>
<td>Order of operations, algebraic expressions, inequalities, verbal and symbolic representation systems</td>
<td>systematic conceptions and representations, questioning techniques, handling errors in student presentations</td>
</tr>
<tr>
<td>Slippery Cylinders</td>
<td>measurement, volume and surface area of cylinders, square roots, finite and infinite measures, estimation</td>
<td>counterintuitive thinking, student investigations, use of manipulatives, gender issues, group work</td>
</tr>
<tr>
<td>The Marble Line</td>
<td>discrete and continuous variables, modeling linear functions, interpreting slope and intercept in context, rate of change, absolute change</td>
<td>notion of variable, checking for understanding, heterogeneous grouping, questioning techniques, representational systems</td>
</tr>
<tr>
<td>Root of the Problem (unpublished)</td>
<td>radical expressions, factoring quadratics, solving radical equations, approximation, rational and irrational numbers</td>
<td>addressing student errors, assessing prerequisite knowledge, on-the-spot change of lesson plan, handling errors in student presentations</td>
</tr>
<tr>
<td>The More Things Change</td>
<td>exponential growth, evaluating exponential expressions, instantaneous and average rates of change</td>
<td>listening skills, small-class activities, discussion techniques, gender issues, competitive students</td>
</tr>
<tr>
<td>What is π Anyway?</td>
<td>area and circumference of circles and annuli, approximation, definitions of π and infinity, ratios without units</td>
<td>assessment, use of journals, grading procedures, effectiveness of models, representations for infinity</td>
</tr>
<tr>
<td>Ships in the Fog</td>
<td>parametric equations, minimization, systems of linear equations, scaling</td>
<td>confusion about units and scale, questioning techniques</td>
</tr>
</tbody>
</table>

Table 1: Case titles and content, in the order discussed in the combined course.

cylinder by rolling the paper and taping together either the long edges or the short edges. Students responded individually, in writing, to these four prompts:

- If you were to pour puffed wheat into each of the cylinders your group just created, which cylinder would hold more puffed wheat?
- How could you determine the correct answer and convince someone else of its correctness?
- What understandings would someone else have to have in order to follow your argument or demonstration?
- What connections does this question about cylinders have to other areas of mathematics?

Students were permitted to leave as soon as they handed in written responses to all four prompts, and were expected to read the case carefully prior to the next class meeting. The post-case discussion contained four prompt categories: mathematical issues, analysis of student thinking, pedagogical issues, and contextual issues. Groups were required to select one choice from each category. One member of the group served as the recorder for each category, writing down as complete notes of the discussion as possible, including who made each contribution to the discussion. Initial concerns that the recording student might be less engaged in the discussion were generally unfounded. In fact, the recording group member often stopped the group discussion to catch up and then voice an opinion. Recording duties rotated with each category so
that every group member served as recorder once during any case discussion.

The mathematical prompts for this case discussion were:

- What role do units and dimensions play in this problem? How well does each group of students understand the units and dimensions involved in the calculations and physical model? Explain.
- Answer the extra credit question posed by Mrs. Lister on page 32, right before the start of the “Lucy’s Work” section. Which of the named students from this case do you think could solve this successfully? Which could not? Explain.
- Address Sergio’s question from page 33. You may use a calculus argument for your own understanding, but also discuss what experiments could be done in Mrs. Lister’s class to help her students reach an answer to Sergio’s question.

The prompts for assessing student thinking were:

- LaShauna makes a comment on page 32 about the area of a square and a circle with the same perimeter. On that same page, Kelly makes a comment about pipes with different diameters. Both students are making connections to previous work. What are the similarities and differences in the prior learning they are trying to transfer?
- Analyze Lucy’s work. What mistake(s) did she make? Why? What would you say to her?
- For each of the named students in the case, discuss what he or she learned or failed to learn about the relationship of perimeter and area to surface area and volume. Support your claims with evidence from the case.

The pedagogical prompts were:

- Did Heather ask the students to generalize too soon? Explain and support your answer with evidence from the case
- Are the manipulatives helpful in understanding and solving this problem? Are they always useful? Is hands-on learning always helpful? Support your answer with evidence not only from the case but also from your own experience as mathematics students and teachers.
- Heather was shocked to find out that her students did not understand the relationship between surface area and volume. She was also surprised to find out how tentative at least one student’s understanding of exponents and square roots was. How might Heather have double-checked her assumptions about her students’ understandings?

The contextual prompts were:

- The boys in this case seem to be much more confident than the girls. It also seems that they are more strident about being wrong, or not knowing how to solve the problem. Are the attitudes represented by these groups of boys and girls typical of what you may have observed in other math classes? What accounts for the differences? How do you think this affects attitudes and learning in mathematics?
- The seed of this idea was planted by a teacher colleague with whom Heather seems to have a good working relationship. What factors do you think help or hinder such collegial relationships between teachers? If you are a new teacher, how can you develop such relationships from the start?
After the Combined Course

Subsequent to the combined course, during the second semester of the first year, one GTA was again assigned to precalculus. Two served as GTAs for other lower-division courses and one was teacher of record for a section of Business Calculus. Two participated in a weekly teaching seminar. The other two engaged in no regular, organized discussion of teaching mathematics. In the fall semester of the next year in graduate school, three were teachers of record: two taught a single section each of Long Calculus I (the first half of a mainstream Calculus I course). One taught three sections of Business Calculus I and the fourth moved to a research assistantship.

Data Collection and Analysis

In this section, we discuss several types of data collected, and what we found within each type of data. In the discussion section, we integrate the data to create a more holistic picture of each GTA participant.

Case Artifacts

Analysis of the case artifacts is based on Beijaard, Verloop, and Vermunt’s model of teacher identity from a personal knowledge perspective. They divide teacher identity into three poles as shown in Figure 2. Subject-matter expertise is knowledge of the content of the course. Didactical expertise is the ability to plan, conduct, and assess a class session. Pedagogical expertise in this context includes such aspects as supporting the psychological and emotional well-being of the students, engaging students in the learning process, and adapting to meet the needs of individual learners. Individuals are located within the framework based on the relative importance they assign to each of these types of expertise.

Figure 2: Beijaard, Verloop, and Vermunt’s model of teacher identity through a personal knowledge perspective.

Early career secondary mathematics teachers typically cluster along the pedagogical/didactical axis, with experienced secondary teachers moving towards the center of the triangle. Mathematics graduate programs traditionally take the implicit view that a subject-matter expert with minimal on-the-job training will be an effective teacher. To the extent that additional training is provided for mathematics GTAs, that training falls largely in the didactical branch of this model. Thus, most research mathematicians and mathematics GTAs likely fall along the subject-matter/didactical edge of Beijaard’s triangular identity model.
Rather than ask participants to rank-order statements that might be construed as aligning with a particular expertise, we coded their comments during post-case discussions as being aligned with one of the three poles: subject-matter, didactical, or pedagogical. See Figure 3 for an excerpt from one case discussion and how comments were coded; this excerpt is in response to the second “assessing student thinking” prompt from the Slippery Cylinders case.

The relative proportion of each type of comment situates the discussion within the framework during each of the case arcs. In particular, it provides a visual representation for each participant of their movement (or lack thereof) towards the more central location occupied by experienced secondary mathematics teachers. To clarify how each participant was located in the framework based on their comments during a single case discussion, we provide an example. Suppose the count was six “subject-matter” orientation (S) comments, three “didactical orientation” (D) comments, and two “pedagogical orientation” (P) comments. Comparing categories S and D, 6 of the 9 comments were in category S, so we place a mark $\frac{2}{3}$ of the way along the SD side of the triangle, closer to S. Similarly, we place a mark $\frac{3}{4}$ of the way along the SP side of the triangle (closer to S) and a mark $\frac{3}{5}$ of the way along the DP side of the triangle (closer to D). We then construct the triangle created by those three marks. The participant’s location within the framework is at the incenter of the constructed triangle, where the three angle bisectors meet (see Figure 4).

It is within the framework of the discussion prompts that we analyze the identity locations of each participant and construct an identity path. As noted, we did not reach a full case discussion for
Ships in the Fog. The identity paths for the remaining six cases for each of the four GTA participants are shown in Figure 5. It is important to note that these are extracted comments from group discussions. As such, they represent only a fragment of the entire discussion in which the subject was involved for each case, but they critically represent the areas in which the participant was willing to voice his or her view and make a contribution. Thus, they provide interesting insight into the “comfort zone” of each participant over the duration of the combined course. In contrast, a similar construction for one of the senior mathematics education majors is shown in Figure 6. For brevity, we do not include the other 17 identity paths, but the one shown is representative of the remaining undergraduates.
Survey

All 23 of the students in the combined course provided survey responses and course artifacts used in the study. On the first and last day of the combined course, students completed a 101-item survey using forced-choice Likert-like items. The four response categories were “strongly agree,” “agree,” “disagree,” and “strongly disagree.” Each category score was translated to a numerical score of “1” for “strongly agree” up to “4” for “strongly disagree”. For positively correlated items, score change was calculated as pre-test score minus post-test score. Negatively correlated items were reversed: score change was calculated as post-test score minus pre-test score. Thus, for a positively correlated item such as “College math professors are expected to teach well” a student who agreed (2) on the pre-test but strongly disagreed (4) on the post-test would have a score change of $-2$. The same pre/post responses for a negatively correlated item such as “I would like a job in which I don’t have to teach” would receive a score change of $+2$.

The items were divided among three subscales:

- Mathematician identity (43 items). These questions are intended to elicit the extent to which the participant identifies as a mathematician either currently or as a future goal. The items in this subscale were closely adapted from the Mathematics Attitude Inventory\(^{31}\) (MAI) and loosely adapted from FICSMath\(^{26}\). The MAI was previously validated for preservice elementary mathematics teachers; items were modified to reflect secondary and post-secondary topics. FICSMath was validated for use with college freshmen across disciplines\(^{13}\). Positive score change within this subscale indicates a movement towards a stronger view of oneself as a mathematician.

- Epistemological beliefs and attitudes (25 items). The items in this subscale are intended to elicit participants’ views on the nature of mathematics and knowledge through the lens of views on effective mathematics instruction. These items were taken from a survey instrument previously validated for preservice elementary mathematics teachers\(^{25}\) and modified to reflect secondary and post-secondary topics. Positive score change within this subscale indicates a movement towards a more constructivist view of mathematics and the teaching of mathematics, i.e. towards a belief that students can construct their own understandings of mathematics and that effective instruction capitalizes on that.
• Teacher identity (33 items). The items in this subscale are intended to elicit the extent to which the participant identifies as a teacher, either currently or as a future goal. As with the “mathematician identity” subscale, these items were adapted from the MAI and from FICSMath. Positive score change within this subscale indicates a movement towards a stronger view of oneself as a teacher of mathematics.

Each of surveys from which items were taken intact or adapted slightly had previously undergone test-retest validation. However, the source instruments were validated for populations other than the subject population, so drawing conclusions about identity or epistemological beliefs from a single survey administration alone should be taken as tentative. An item-by-item change analysis is beyond the scope of this paper; we include only results for each subscale as a whole. These results are intended to provide general insight into shifts over time, rough comparison between the two populations, and triangulation with case artifacts and student performance data.

We collected pre- and post-survey data for all four GTA participants and for 17 of the 18 undergraduate pre-service teachers. In the absence of validation for the GTA population, we cannot make assumptions of normality to allow for a two-sample \( t \)-test comparing mean changes between the two populations. Nor, given the small sample size for the GTA population, can we use a two-sample \( z \)-test. Thus, we cannot test for statistically significant differences between the mean changes in subscale score for the two populations. However, we can make some observations using descriptive statistics, so we present here the box-and-whisker plots for mean change in each subscale for both populations.

![Box-and-whisker plots for mean change in each subscale for both populations.](image)

Figure 7: A positive change indicates a shift towards a stronger mathematician identity.

Both populations were taking mathematics courses at a higher level than in their previous experiences. All of the undergraduates were taking either abstract algebra or advanced calculus for the first time; some were taking both. The graduate students were taking graduate level mathematics courses for the first time. Although the mean changes between the two populations are close, we note in Figure 7 that while somewhat more than a quarter of the undergraduates had a shift towards a stronger mathematician identity, none of the graduate students did.
The undergraduate students had previous experience in secondary mathematics education coursework presented with a constructivist view. Their mean change in epistemological beliefs was lower than that of the GTAs, as shown in Figure 8. In addition, 75% of the GTAs displayed a greater shift towards a constructivist view of mathematics and teaching than the mean and median change for the undergraduates.

It is in the teacher identity subscale that we see the most striking difference between the two populations. The undergraduates had a median change of zero, indicating no change in their teacher identity. The data for the undergraduates is nearly symmetric, with one slight outlier dragging the mean slightly below the median. The GTAs demonstrated a striking shift away from a teacher identity, with their entire interquartile range falling in the lowest quartile of the
undergraduate population. While this is disheartening, consideration of this shift within the context of case arcs and reflective writings presents a more complete picture for each individual, and is addressed in the discussion section.

**Student Performance**

All introductory mathematics courses at the participant institution are closely coordinated, with common midterm and final exams. We compare student performance in the participants’ sections to student performance in sections taught by other GTAs in their first teaching experience, as well as to overall student performance in all sections. This quantitative data contributes to triangulation with respect to participants’ self-perception of teaching competence. It also provides a small degree of insight into effectiveness of the use of case study for professional preparation during the first year of graduate studies.

There are unfortunate concerns regarding the fidelity of the data for coordinated Business Calculus course during the first semester as instructor of record for one of the subjects. A second subject was placed on research assistantship, so we have no data for student performance for that subject.

The remaining two subjects, GTA1 and GTA3, were each assigned to teach one section of Long Calculus I with 45 students each. The content of Long Calculus I is one-third review of precalculus, one-third limits, and one-third introduction to derivatives (up to the chain rule). The course is closely coordinated, with common online homework, midterm exams, and final exam. Exam grading is done collectively, with one grader for the same problem across all sections. The didactical and pedagogical aspects are at each instructor’s discretion, and there is no ongoing instructional support provided to GTAs.

Results from the participant sections and from comparison sections are given below in Figure 10. In this figure, GTA1 and GTA3 are study participants, G1 is a first-semester teacher of record who had not participated in the study activities and L is the average of two experienced full-time lecturers teaching six sections total. Test 1, Test 2, Test 3, and Final Exam are the collectively graded common exams. “Classwork” is the portion of the course grade that is instructor-dependent and based on classroom instructional activities. “Overall” refers to the final weighted course average. It is worth noting that this is a Pass/Fail course that does not affect overall GPA, and students with sufficiently high course averages are exempt from the final. Both of those factors depress final exam averages, particularly in sections with stronger performance prior to the final.

On each of the common exams, the three first-time teachers of record appear to be closely matched, and student performance falls below that of experienced lecturers. Final course averages were significantly higher for the GTAs who participated in the combined course that for non-participants, however, despite the fact that they assigned lower daily grades, the only portion of the course average under their direct control. While this superficially appears contradictory, it actually reflects declining student participation in the section taught by G1. Students who did not take a given exam are not reflected in that exam average but are reflected in the course average, so as some students stopped attempting exams, the exam average for that section increased, but the overall course average dropped. Thus, it can be argued that GTA1 and GTA3 were more successful at student retention and engagement over the course of their first solo teaching
experience than was G1. This may in turn reflect stronger ability to engage students as learners, one aspect of pedagogical expertise. It is also worth noting that the overall student performance in GTA1’s section matched that of the experienced lecturers.

Discussion

Situating and Integrating the Data

Each of the four data types taken separately provides only a portion of the complete picture. Our intent in this paper is to focus on the impact of case discussion within the combined course, with additional data sources serving to situate the case arcs within context and to enrich our understanding of their impact. Indeed, the survey data provides snapshot bookends within which to triangulate the identity path observed from the case discussions. Similarly, reflective writings provide more nuanced snapshots taken at mid-semester and end-semester. Student performance data gives us no qualitative information, but rather serves as a measure of the mid-range impact of internalized thoughts about teaching that may have arisen from the in-depth case discussions in the combined course.

From these separate measures, we can piece together an idea of the development of each participant separately. Although we cannot distinguish between the impact of the case discussions and the impact of the precalculus classroom experience, we have a proxy measure in the comparison of student performance in the first solo teaching experience. While it might be argued
that the precalculus instructional role, in and of itself, provides a superior first-year preparation to other first-year teaching experiences, that question is beyond the scope of this paper. With the goal, then, of integrating the supporting data with the identity trajectories as measured by case discussion, we look more closely at each participant separately. We present each participant’s identity path again for ease of seeing how the other data fits within this picture.

**GTA1**

- Mean Mathematician Identity Change: $-0.255813953$
- Mean Epistemological Change: $-0.08$
- Mean Teacher Identity Change: $+0.060606061$

GTA1 presents some interesting apparent contradictions. After an initial foray into didactical and pedagogical contributions, she retreated to a purely subject-matter expert role in case discussion and stayed there. Despite this, she showed a movement towards a stronger teacher identity and away from a stronger mathematician identity based on the pre/post-survey. At the same time, GTA1 is the only participant who demonstrated a movement away from a constructivist view of mathematics and towards a view that students should copy worked examples rather than developing their own methods. In addition, this participant is also the only one who self-identified changes in her views of mathematics and teaching during the combined course, writing on her post-survey:

> “Over the course of the semester I’ve become more confident about explaining math. I’ve also come to believe it’s more important to present material in an organic way that allows students to understand where the material is coming from, rather than just memorizing how to do it.”

This apparent contradiction between the student’s self-perception and the quantitative changes is perhaps clarified by a written reflection from this participant during the last week of classes:

> “I learned how important it is to time when to jump in and explain something to a student. When working through problems with students in precalculus, it became clear that if you jump in too early to correct a mistake in the way a student is approaching the problem at hand, you sometimes end up confusing them more because they don’t understand what they’re doing wrong or how it is different from the correct approach. On the other hand, if you wait too long, they get confused and have to completely start over.”

It appears that GTA1 is not wrestling with whether students should be encouraged to develop their own (mathematically correct) approaches, but rather with how to allow that to occur without permitting errors to go unchecked. Her view of teaching and of her own competence as a teacher is clearly in flux. Given that significant shifts in identity and teaching practice occur at the boundary between undergraduate studies and first teaching experience for secondary teachers.
and that the “productive friction” produced at those boundaries is an important facet in developing effective instructional practice,\textsuperscript{29} this actually speaks to strong potential for this participant to develop into an effective teacher. That potential appears to be bearing fruit: GTA1’s student performance as measured by overall course grade is on par with that of experienced full-time lecturers, and higher than that of all other first-time graduate teachers of record for the same course.

\textit{GTA2}

\begin{itemize}
\item Mean Mathematician Identity Change: 0
\item Mean Epistemological Change: +0.28
\item Mean Teacher Identity Change: −0.15
\end{itemize}

This subject remained solidly wedded to the subject-matter/didactical edge of the identity triangle, slowly moving out of the subject-matter corner and towards a more balanced contribution of subject-matter and didactical comments in case discussions. GTA2 indicated on the post-survey that participation in the combined course and teaching precalculus had resulted in no change to views of mathematics or teaching. Indeed, there was very little shift in the subscales. However, in response to a reflective writing prompt in the last week of the semester, this subject wrote:

“\textit{I have learned to not expect students to understand all material leading up to a certain topic. I used to be nervous and race through examples, but I have slowed down and started asking more questions. In doing so I find that sometimes the new material isn’t the problem, but it is one of the foundation pieces giving them trouble} ... \textit{[T]he semester as a whole was very humbling. There were many times in precalculus that a question was asked, and I didn’t have the answer. [Participation in the combined course] allowed me to find help and gain a deeper understanding of specific topics. I am much more adept at asking for help or researching solutions now, instead of just trying to plow my way through and maybe not explain something well, or worse, explain it wrong.”}

The shift towards greater balance between didactical and subject-matter expertise seems reflected in this writing. This subject’s focus is on developing a deeper subject-matter understanding in order to structure lessons and explanations more effectively. There is only minimal attention given to engaging the student as a learner by asking questions. This participant was assigned to teach three 19-student sections of Business Calculus I as his first teaching experience. Again, appropriate data for this course is not available to provide a comparison of his students’ success.

\textit{GTA3}

Like GTA1, GTA3 started out contributing to case discussions in each area of expertise, then retreated to the subject-matter expertise corner. For this subject, however, the retreat was
Mean Mathematician Identity Change: $-0.139534884$

Mean Epistemological Change: $+0.2$

Mean Teacher Identity Change: $-0.212121212$

temporary and GTA3 slowly resumed making contributions from multiple vantages, with an increasingly central location in the identity triangle. Despite a slight shift away from a teaching identity, away from a mathematician identity, and towards a more constructivist view of knowledge and mathematics, GTA3 indicated no self-perceived changes in any of those aspects. GTA3 was the only participant who actively referenced the case studies in response to a reflection prompt at the end of the semester, writing:

“I’m always trying to understand how students think and the misconceptions and common mistakes they have. By reading the case studies and watching/teaching/tutoring/grading for [precalculus], I have learned more misconceptions and mistakes. I can teach kids the theory and the correct way to do stuff, but if they are consistently making more than simple errors, I need to know how to help them. Hopefully, the more experience I get the easier it is.”

Despite a desire to develop pedagogical expertise, GTA3 still felt more comfortable expressing subject-matter and, to some extent, didactical expertise. Although the case arcs were viewed as a valuable companion to the teaching experience and participation in the combined course, GTA3 did not see them as having conferred the skills needed for effective teaching. As a first solo teaching experience, GTA3 was assigned as instructor of record for a 45-student section of Long Calculus I. Student performance in her section exceeded that of the first-time GTA without the combined course and case discussion experience, but fell below that of GTA1. It would be interesting to see if additional experience allows GTA3 to assume a role of pedagogical and didactical expert to balance the subject-matter expertise, and how future sections compare to those taught by other similarly experienced GTAs.

\textit{GTA4}

Mean Mathematician Identity Change: $-0.046511628$

Mean Epistemological Change: $+0.4$

Mean Teacher Identity Change: $-0.424242424$

In many ways, GTA4’s identity path over the course of the case discussions is similar to that of GTA3. An initial willingness to contribute to discussions in all three arenas of expertise quickly changed to a retreat to the subject-matter/didactical axis. In the case of GTA4, however, the
identity location at the end of the semester was less central than at the beginning of the semester, and more closely anchored to the subject-matter corner of the triangle. GTA4 exhibited almost no change in mathematician identity, the greatest shift away from teacher identity, and the greatest shift towards a constructivist view of mathematics and teaching among the four participants. Nonetheless, like GTA2 and GTA3, GTA4 self-identified no change in those aspects over the course of the semester and the response to the reflective prompt centers on reinforcement of previously held views:

“[S]omething that was reaffirmed is that repeatedly going through a lesson is extremely helpful. In [precalculus], going over a topic for the first time, you are bound to make mistakes or forget to mention a small piece of information that may be helpful for a different variation of the problem ... [this] reaffirmed the importance of preparation, especially with practicing the lesson beforehand.”

In addition to a focus on reaffirmation of prior knowledge, this subject’s writing also reflects the positioning in the subject-matter corner and along the subject-matter/didactical axis of the identity triangle. This subject was placed on research assistantship after the first year, so we have no data regarding subsequent student performance.

**Conclusions and Limitations**

There are some limitations to this study. The participants were all native-English speakers who had attended four-year colleges where the institutional focus was on good instruction rather than research. The impact of case discussions in professional preparation of non-native English speakers or of graduate students emerging from undergraduate programs at research-focused universities might be far different. The survey was not validated for this particular population, so the conclusions we can draw from prior test/retest with this group are limited to general insight and triangulation with other data. The pilot group was small, and we only have subsequent student performance data for two of the participants. The cases were drawn from high school classroom contexts rather than college contexts. Although the mathematical content was similar to that of the precalculus course, the case arcs did not allow for discussion of contextual issues relevant to higher education, nor did they address the mathematical content of calculus. For future implementations, we might seek to use cases based on college classrooms, rather than secondary ones.

Nonetheless, there is some degree of evidence that participation in the combined course and in discussion of case studies had a positive impact on the participants’ views of the nature of mathematics and effective mathematics instruction, and on their students’ subsequent performance in the first solo teaching experience. This is concurrent with a reduction in the extent to which they view themselves as teachers and as mathematicians.

After the first case discussion, all four participants retreated to a purely or primarily subject-matter expert role. With the exception of GTA1, they then moved back to a more central location, with a balance of contributions from each of the three positions of expertise. In all four cases, however, the GTAs remained lodged closest to the subject-matter expert corner of the triangle. Given the nature of the combined course, this is not entirely surprising. In each of the
case discussions, they were the figures with the most mathematical authority, having completed an undergraduate degree in mathematics and having begun graduate studies in mathematics. They also had the least pedagogical authority, as the other group members had completed considerable amounts of education coursework and were concurrently enrolled in a mathematics pedagogy course. At the beginning of the semester, neither the GTAs nor the undergraduates had much didactical authority, not having planned or conducted class sessions. As the semester wore on and the GTAs gained experience delivering mini-lessons in the precalculus classes, their willingness to assume didactical authority in the group discussions shows up as a shift towards the didactical corner of the triangle. It is unclear whether the pedagogical richness added to the conversations by the secondary mathematics education majors was an added value for the GTA participants, or whether the GTAs would have been better served by participating in case discussions in which they felt more comfortable expressing and exploring their pedagogical views rather than deferring to those with greater perceived expertise. In future implementations, we might seek a more even balance between senior secondary mathematics majors and graduate students, or we might consider teaching a case-based professional development course exclusively for mathematics graduate students but supplemented by readings on mathematical pedagogy.

Three of the four participants identified less closely with teaching at the end of the combined course. Those same three participants had a shift towards a more constructivist view of mathematics and knowledge. Their reflective writing indicate that these two shifts may be linked. They saw themselves as less competent at teaching than they did at the start of the semester, and they place greater importance on understanding how students are thinking about mathematics as a central facet of being a good teacher. These shifts, then, may be considered as an overall positive within the context of helping graduate students develop effective teaching strategies and ways of understanding of students as learners.

Student performance in coordinated courses taught by the participants exceeded student performance in the same courses taught by graduate students with the same level of teaching experience and, in one case, matched that of experienced full-time lecturers teaching the same course. However, we note again that we only have reliable data for student performance for two of the four participants.

Overall, we find the use of case discussions in the preparation of mathematics graduate student teachers to be promising. Cases allow for fruitful dialogue focused on each of subject-matter, pedagogical, and didactical expertise. They also provide a vehicle for examining teaching practice and student thinking in a meaningful context but without the immediate pressure or personal jeopardy of a current teaching assignment. Most promisingly, case discussions establish a forum for elucidating specific aspects of teaching expertise in order to accelerate first-year graduate students toward the more balanced teaching identity exhibited by experienced teachers.
References


