



## Trending Mistakes in Signals and Systems courses

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## **Work in Progress: Trending mistakes in Signals and Systems courses**

### **Abstract**

Signals and Systems is a core course in undergraduate electrical engineering curriculum. The concepts taught in this course become foundational knowledge for many advanced courses, which necessitates conceptual understanding of the topics in this course. Despite many attempts to make this course easy to understand for students, its conceptual understanding remains a challenge. The objective of this study is to identify students' understanding and retention of signals' representations, operations, and transformation in and between frequency and time domain by identifying the mistakes that undergraduate electrical engineering students continue to make even after repeatedly applying the concepts in advanced courses. For this study, three questions related to the basic concepts (drawing of a sinusoidal signal, and Fourier analysis) in signals and systems courses were given in the midterm exam of undergraduate electrical engineering students while they were taking signals and systems course. A year later, the same three questions were given to the same students in their digital signal processing midterm exam. All students were taught by the same instructors. The objective of taking the same exam of the same students twice with a gap of one year was to identify how the understanding and retention of students develop as they continue to encounter the same concepts repeatedly in different contexts. The results reveal that for two out of the three questions given, more than fifty percent of the students who solved the questions (partially or completely) correctly for the first time did not retain the understanding a year later. For the third question, the percentage of the students who retained the understanding was greater but still comparable with the ones who forgot the concept a year later. This study identifies some very basic and important Signals and Systems related concepts that are hard to understand and retain and can help to suggest pedagogical techniques to improve students' understanding.

### **Introduction and Background**

Engineers use mathematics and science as tools to build products that serve the society (Kirschenman and Brenner, 2009). As just an end user of these mathematical equations and models, sometimes, engineering students grapple with conceptually understanding concepts that are abstract or intangible (Sazhin, 1998; Bingolbali, Monaghan, & Roper, 2007). Therefore, engineering courses that have mathematics intensive topics can become a challenge to learn well and Signals and Systems (SS) is one of these courses (Chi, 2005). This course includes cognitively challenging topics like complex equations, complex domains, representations of same signal with different equations in different domains, abstract signals, drawing of abstract signals (Fayyaz, 2014). The even bigger challenge for an engineering student in conceptually understanding these concepts is to translate and apply these abstract concepts on real life signals and real life systems (Nasr, Hall, & Garik, 2005; Ferri et al., 2009; Han, Zhang, & Qin, 2011; Tsakalis et al., 2011; Fayyaz, 2014).

Many qualitative (Fayyaz, 2014) and quantitative (Huettel, 2006; Ogunfunmi, 2011) studies have been done in understanding why these concepts are difficult to understand, and in developing successful pedagogical strategies for teaching these courses (Wage, Buck, and Hjalmarson, 2006; Padgett, Yoder, & Forbes, 2011; Fayyaz, 2016). Nonetheless, the concepts taught in this course

have remained cognitively challenging for students (Fayyaz, 2016). The topics covered in SS courses are important to learn well as these become foundational concepts for many advanced courses like image processing and communication.

This longitudinal study is an attempt to identify how students understand and retain the graphical representation, Fourier transform, and inverse Fourier transform of a sine signal with a phase shift. There have been many attempts in the past in understanding students' difficulties in basic SS related topics. This study is important from two aspects. Firstly, there have not been many longitudinal studies done in understanding electrical engineering students' conceptual learning of SS related concepts. Longitudinal studies are important because they help us identify the concepts that if not initially learned, gets better with time as students keep encountering them repeatedly. In addition, longitudinal studies help to highlight concepts that if not learned properly are hard to learn at later stages as well. Secondly, this study is conducted in Pakistan where not much of this kind of research is conducted already. It is important to identify students' difficulties in different educational settings. Similar learning difficulties observed in more than one educational settings highlight an issue that requires serious attention. Additionally, if different learning difficulties are observed in different educational settings, then the differences in the pedagogical techniques in these settings can be compared to improve overall learning.

## **Method**

One of the major challenges in understanding SS related concepts is the ability to visualize one single signal simultaneously in both time and frequency domains (Fayyaz, 2014). The questions used for this study aimed to observe students' understanding and retention of transformation and drawing of same signals in different domains. The signal chosen is a shifted sinusoidal signal because it is a basic (also most encountered) signal and the author considers that understanding the basics of SS related concepts on a basic signal is the most important step in understanding the whole course.

For this study, three questions were used. The questions were adapted from a qualitative study (Fayyaz, 2014). For the qualitative study, these questions were designed after piloting in various stages. To ensure that no new questions are unnecessarily designed, during the pilot stages of designing the questions, a detailed quantitative analysis of students' responses to different questions in Signals and Systems Concept Inventory (SSCI) (Chi, 2005) was conducted followed by interviewing students for understanding the reasons behind the most frequently incorrectly answered questions (Fayyaz, 2014). The questions finalized for the qualitative study (none from SSCI) were the ones that best facilitated accessing students' thinking and were considered important concepts to understand after consultation with the experts (Fayyaz, 2014).

In the first question, students were given two impulses in time domain, which made a pure sine signal in frequency domain. Since students generally encounter a sine or cosine in time domain with two impulses in frequency domain, this question aimed to see whether students mentally adhere to the visualization of a sinusoidal graph in time domain with their respective Fourier transform as impulses or if the students can manage to conceptualize the reverse as well.

In the second question, students were given a signal which was a sum of a time-domain sine function (with certain frequency and phase) and a constant complex exponential. Students were asked to draw the Fourier transform of the signal. This question aimed to see how students understand the Fourier transform of a phase shifted sine (including  $-\pi/2$  phase shift with respect to a pure cosine) function. This question also intended to understand how critically students look at and understand a complex exponential function. Additionally, this question explored students' understanding of the linearity property of the Fourier transform and how students add and draw the sum of Fourier transforms of two individual signals.

In the third question, students were asked to just draw and completely label a phase shifted sinusoidal signal. The objective of this question was to observe how students relate the concept of phase of a sinusoidal signal and time shift of a sinusoidal signal. Furthermore, this question intended to observe how students understand the combined operations of scaling and shifting a signal in time.

The above-mentioned three questions were given in the Signals and Systems Mid-term exam to junior year undergraduate electrical engineering students in Fall 2016. These questions were given in the Mid-term exam as students are expected to be more prepared and effective on the exam day. In Spring 2017, these students took a course on Analog and Digital Communication which is heavily dependent on SS related concepts. In Fall 2017, these students (now in senior year) were enrolled in Digital Signal Processing (DSP) course and these same questions were given in the Mid-term exam of DSP. Sixty-three students were common in the pool of students who answered these questions in Signals and Systems Mid-term exam in Fall 2016 and then in DSP Mid-term exam in Fall 2017. The data is analyzed and reported for these sixty-three students. These students were taught Signals and Systems and Analog and Digital Communication course by one instructor. Another instructor taught DSP. Each course was taught to all the students in this study by the same instructor which means the instructional format was uniform for all students.

This is a work-in-progress (WIP) paper. The data collected is very rich and has a huge potential for presenting possible reasons for hurdles in students' understanding of these concepts. For example, the students' responses highlighted a mistake that was not anticipated before the study. The students were having difficulty in finding the phase of a shifted sine signal because they were applying time shift property to find the phase. The time shift property gives the expression of a linear phase and that when multiplied with an impulse, gives just a single phase. Students were missing the step of multiplication with an impulse and so got confused in getting the correct result. Many students who could not find the phase of a shifted sine signal made this same mistake. The detailed analysis of all mistakes of students in each question can possibly bring out more surprises like these. For the scope of this WIP paper, the students' responses are categorized in four groups. First is the category of students who attempted the question correct or half correct on both the first and second exams. For the scope of this study, these students are assumed to have understood and retained some if not all of the concepts covered in that particular question. Second category is students who attempted a particular question either completely or partially correct in the first exam and gave a completely incorrect solution in the second exam. For the scope of this study, these are the type of students who are deemed most important to identify because (given the teaching methodology was same for all the students)

their learning behaviors, ways of understanding, and thinking processes are such that once learned a basic concept, despite encountering it repeatedly in related contexts, they are unable to retain it. The third category of students is those who do not get it in the first exam and later in the second exam they are able to completely or at least partially solve the question correctly. This study considers this group of students important because getting an insight on the learning behaviors, ways of understanding, and thinking processes of this group can help to identify the effective pedagogical strategies to help students (that are struggling to understand these concepts) get better understanding of SS related concepts. The fourth group of students is those who solved a question incorrectly in both the exams. The importance of the identification of the size of the first and fourth group of students is in the continuous quality improvement processes of the pedagogical techniques being employed for SS and related courses. A half-correct solution is not a completely correct solution but is not entirely incorrect as well. Author acknowledges that there is rich data hidden within the half correct solutions and actually half correct solutions can lead to understanding what stops a student from correctly solving or understanding an SS related question. The exploration of these details are out of the scope of this WIP paper.

## Results

The results of students' performances in each of the questions is shown in Figures 1, 2, and 3 below.

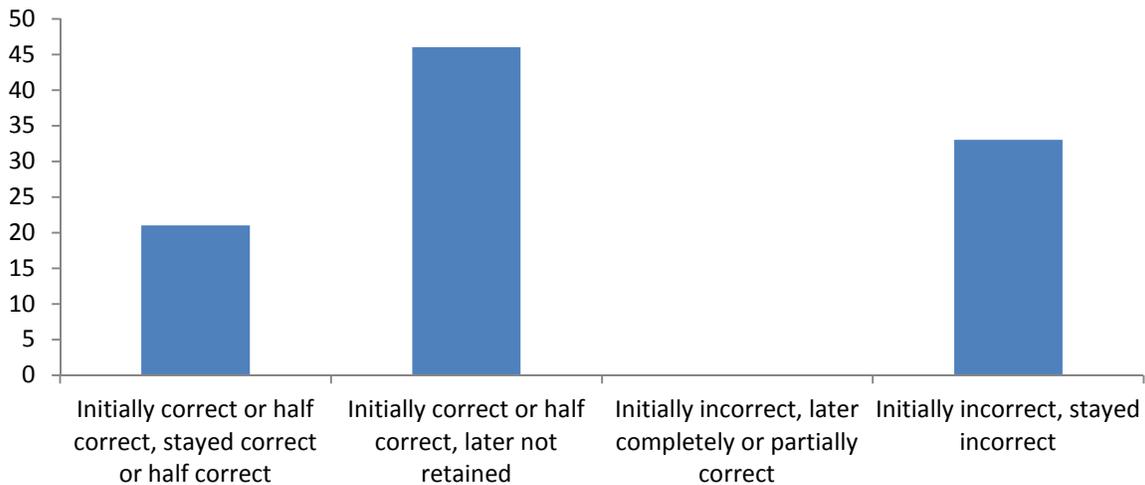


Figure 1. Percentage of students with respect to their performance in Question 1

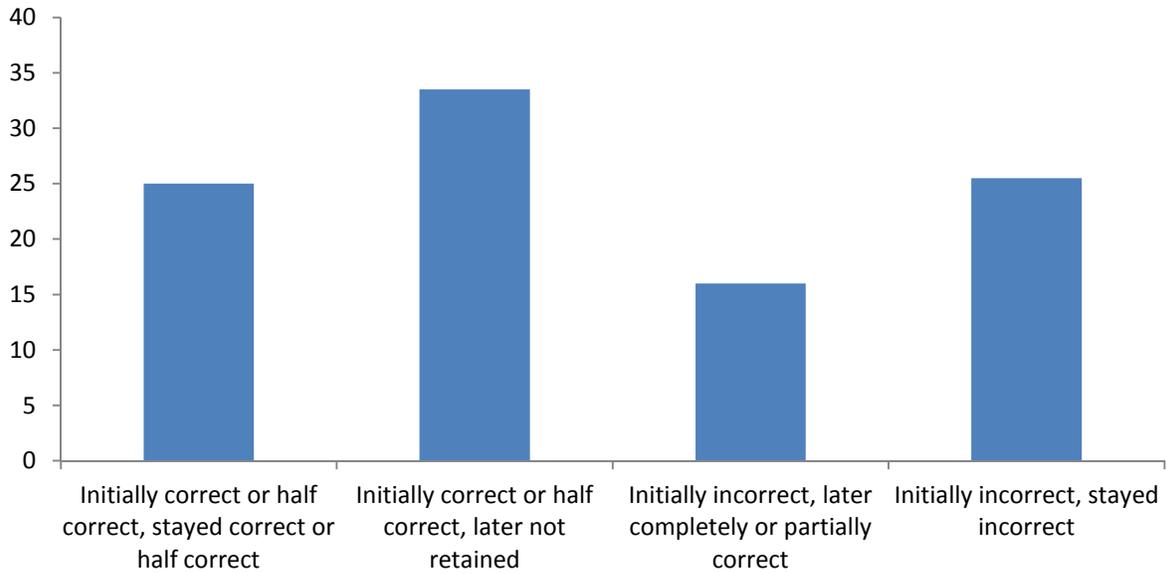


Figure 2. Percentage of students with respect to their performance in Question 2

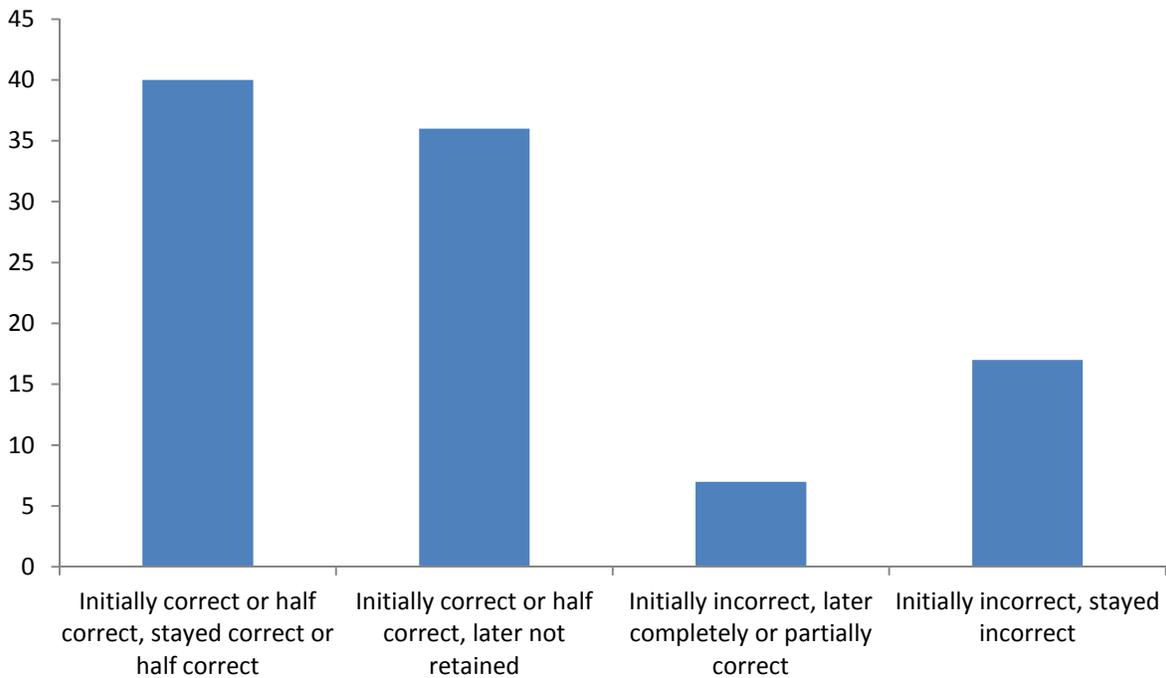


Figure 3. Percentage of students with respect to their performance in Question 3

A comparison of students' overall performances after a year of taking SS course is presented in Figure 4. This result is obtained by considering a student's performance as *good* if the student answered two or more (more than half) questions correctly or half correctly in one exam, and as *poor* if the student answered two or more questions completely incorrectly in an exam. A *consistent good performance* means that the student's performance was *good* in both the exams. A *consistent poor performance* means that the student's performance was *poor* in both the

exams. A *declined performance* means that a particular student's performance was *good* in the first exam and *poor* a year later in the second exam. An *improved performance* means that a particular student's performance was *poor* in the first exam and *good* a year later in the second exam.

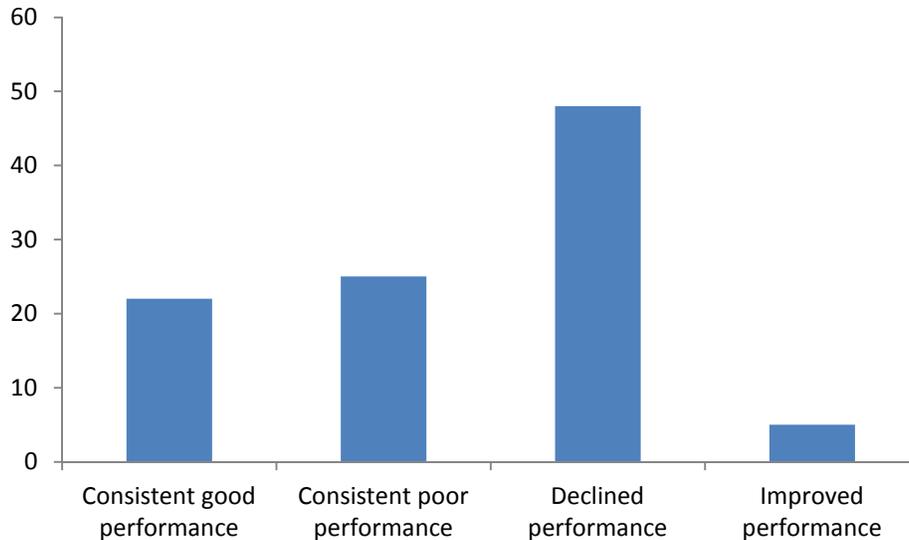


Figure 4. Percentage of overall (all three questions) performance of students a year after taking an SS course

## Discussion

The main objective of this study is to identify the retention of the basic concepts related to SS courses in undergraduate electrical engineering students as they proceed forward in academics. Figure 4 highlights that overall 22% of the students retained and 5% gained better understanding of the basic concepts over the year, which are encouraging percentages for educators. However, the study also highlights an important number 48, which is the percentage of the students that exhibit a decline in their understanding one year after taking SS course. This means no retention. This is a critical information that calls for more research in learning and pedagogy of SS related courses. As the results of this study comes from a very small sample size, and more importantly the sample is from only one institute and one country, these results may not be a representative of the overall student population in different institutions in different countries. The author's next step to validate the results of this study is to repeat this study in a variety of educational settings and compare the results. This comparison will help in two ways. Firstly, this will help to identify the problems in learning that are localized and those that are more general. Secondly, once the localized problems are identified, then the pedagogy of more successful educational settings can be used to inform the design of better pedagogical techniques for this course.

Besides highlighting the retention issues, this study also pinpoints some possible concepts that if not completely understood can hinder a student to solve SS related questions. The identified concepts whose lack of understanding hindered students from correctly solving the questions were linearity and time-shift properties of Fourier transform, and also the difference between the exponential function and complex exponential function.

An example of students' lack of understanding of the application of linearity property of Fourier transform was observed when students were asked to find Fourier transform of  $e^{j\frac{\pi}{3}}$ . Students considered this constant exponential as a constant multiplier and just like a constant multiplier appears as such in the Fourier transform, students incorrectly said that the Fourier transform of  $e^{j\frac{\pi}{3}}$  is still  $e^{j\frac{\pi}{3}}$ . The understanding of the concept of a complex exponential in general seemed weak. In addition to finding incorrect Fourier transform, it was observed that students treated  $e^{j\frac{\pi}{3}}$  as  $e^{j\frac{\pi}{3}t}$  and in some cases as  $e^{\frac{\pi}{3}t}$  or  $e^{\frac{\pi}{3}}$ .

An example of the weak understanding of the application of the time-shift property of Fourier transform was observed when students tried to find the phase of  $\sin(\frac{\pi}{6}t + \frac{\pi}{6})$ . Students have already memorized the Fourier transform of a sine signal and they tried to apply time-shift property of Fourier transform to find the phase of the sinusoidal signal. The application of the property resulted in  $e^{j\omega}$  in the Fourier transform. This  $\omega$  in the exponential term confused students to further simplify for phase. Students did not think of the multiplication property of an impulse function, got stuck at this point, and left the question unsolved.

In addition to identifying the shortcomings in students' conceptual understanding, this study also identifies some practices of solving SS related questions that facilitate students in solving questions correctly.

An example of a successful question solving strategy which most of the students employed to find the phase of  $\sin(\frac{\pi}{6}t + \frac{\pi}{6})$  is that the students first converted the whole expression in exponentials using inverse Euler identity and then simplified. It was observed through students' responses that simplifying a  $j$  with  $e^{j\frac{\pi}{6}}$  to find actual phase shift of  $\sin(\frac{\pi}{6}t + \frac{\pi}{6})$  with respect to a pure cosine was a challenge for most students. This actually brings forward an interesting suggestion. Maybe books should stop using both sine and cosine to express a sinusoidal signal. They may only use cosine and everything else as a shifted cosine. This might make at least one concept a bit straight forward to understand and retain.

Another example of a successful question solving strategy was observed in drawing (and labeling) the time-domain graph of a shifted sinusoidal signal,  $s(t) = \sin(\frac{\pi}{2}t - \frac{\pi}{4})$ . Most of the students who correctly drew the signal followed the rules of combined operations of time-scaling and time-shifting. The students who could not draw and label the graph properly showed a variety of difficulties. Some struggled in distinguishing between the time-shift and phase-shift of a sinusoidal signal. Others first drew and labeled  $\sin(\frac{\pi}{2}t)$  and then shifted the signal by  $\frac{\pi}{4}$  units in time, which is against the rules of combined operations of shifting and scaling a signal.

## Conclusion and Future Work

A very important and rather sad observation in this study is that there is an overall decline in students' performances in SS related questions as they move forward in their degree program despite recurring exposure to these concepts. This definitely begs for attention towards

developing improved pedagogical strategies for SS related courses, which is not possible without an enhanced understanding of the exact reasons for students' struggles in SS related questions.

This study is expected to be repeated in a variety of educational settings in different countries to have a broader understanding of students' learning and retention of SS related concepts. Conducting the same kind of studies in more than one educational settings may provide a deeper insight of students' struggles in understanding and retention of SS related concepts. Additionally, further continuation of this study will be to analyze students' responses more deeply to observe the question solving patterns when a student correctly or incorrectly solves a particular question. Once a richer understanding of the educational settings and patterns in which students solve the questions is gathered, the study aims to suggest pedagogical improvements in SS related questions.

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