How is Calculus Applied in Engineering Statics?

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Where does the calculus go? A follow-up investigation of how calculus is used in core engineering coursework

Abstract

Engineering students must complete long chains of prerequisite courses to proceed through their curriculum. Prerequisite chains of mathematics courses must be completed to proceed into core engineering coursework. How necessary are these prerequisite chains? How much of this mathematics that is taught in a mathematics course reaches application in the following engineering course? This study examines how mathematics is applied in Statics. We apply the mathematics-in-use technique to examine which calculus skills and concepts are applied in this core engineering course. In a way that is surprising giving their dense prerequisites, the amount of calculus needed in statics is just 8% of the course. Despite having long prerequisite chains full of advanced calculus, require just a little bit of basic freshman calculus.

Introduction

“To be effective and useful the design of mathematics courses for engineering students must involve a continuous and informed dialogue between engineering and mathematics departments to which each must contribute fully. The process of dialogue is essential since neither must be the dominant partner. The difficulties usually arise not in deciding what is to be taught but how and at what level. This is where the engineering department must have a clear understanding of what is needed and be able to communicate this effectively to the mathematicians.”

---J.O. Scanlan, 1985

Engineering departments are becoming increasingly concerned about retention and graduation rates as industry in the United States demands more engineering graduates to meet expected engineering job growth in coming decades [1]. However, since many students dropout of engineering, too few engineering students graduate to join industry. [2]. More specifically, most students drop out of engineering not because they failed an engineering course, but because they failed a mathematics course [3-9], with some programs blaming mathematics courses for a third of their dropouts [2; 10-14]. Most engineering programs require a strictly ordered prerequisite “calculus sequence” of Calculus I, Calculus II, Calculus III, Linear Algebra, and Differential Equations. Students must pass prerequisite mathematics courses from the calculus sequence to continue into core engineering coursework such as statics, dynamics, circuits, and thermodynamics. [9; 15-18]. Due to the length of these prerequisite chains in the “math-science death march” [19], engineering students may not take their first course with engineering faculty until their sophomore or junior year [20, 21]. These prerequisite mathematics courses often have
high DFW rates [22-26]. The strictness of this prerequisite chain can particularly hamper female and minority students [21] and students who are already disadvantaged due to disability or lack of access to high school calculus [27]. Students who do not start calculus-ready or fail a course in the calculus sequence may struggle to complete an engineering degree before financial aid runs out.

Because the calculus sequence has such a strong impact on engineering graduation, we must justify these high-failure prerequisite mathematics courses. Reduction in long prerequisite chains is desirable, since inflexible long prerequisite chains and hurt disadvantaged students. [21]. This study maps the knowledge learned in Calculus to when that knowledge is used in core engineering courses.

Background

Engineering students must pass long, consecutive strings of prerequisite courses in engineering curricula, failure in any of which delays the students progress by a semester. For example, a calculus-ready student who takes the required junior-year course “Aerospace Dynamical Systems” must begin with a bi-pronged prerequisite chain (Figure 1). A failure or disruption in any of the 7 courses in the sequence has severe consequences for the student’s graduation time and probability of graduating at all [21]. Can prerequisite structures be modified to include fewer stumbling blocks that may delay graduation in order to improve engineering retention and graduation rates?

Figure 1: The prerequisite relationships at our institution leading to one particular junior-year required course in Aerospace engineering.

Studies have shown that prerequisite mathematics course performance usually has a moderate Pearson correlation (r=0.4 to 0.7) with subsequent engineering course performance [28-32]. Given prerequisite structures, we might assume that the content of these mathematics courses is strongly linked to that of subsequent engineering courses. But is it?

Ideally, the preparation from prerequisite structures allow an engineering professor to pick up where the mathematics professor left off, applying the knowledge of the previous course. A successful handoff requires that students will both be able to recall the mathematical knowledge from the prerequisite course and apply (in the cognitive jargon ‘transfer’) that knowledge to
engineering. Unfortunately, students often forget content that is not revisited in a year [33-38], and cannot transfer what they do remember without special prompting [10; 39-44] As a result, prerequisites do not effectively prepare students, and engineering faculty often reteach mathematical content that has been forgotten. Given these issues with recall and transfer, I question whether engineering students are getting the right mathematical content at the right time [18].

Previous literature has examined the connections between mathematical content in the engineering curriculum [39; 45-48]. Overall, these studies agree that only a small portion of the mathematical content in the calculus sequence is actually applied in engineering courses. Furthermore, application of mathematical content may be separated from the prerequisite course by a year or more, and some applications are taught before the underlying mathematics in sequence. However, this literature depends on faculty self-report, which is often unreliable because a professor’s perception often fails to match their practice [49, 50]. Because faculty self-report may not reflect student experience, I argue that research into curricular alignment requires an alternate means of inquiry. In this study, I choose to examine homework as a course artifact, because homework is the assessment that students engage with most often and homework has the most diverse content coverage.

Previous research on application of calculus in engineering homework problems reveals mathematics in Calculus courses differs greatly from that found in engineering courses. Students may not recognize that the integration encountered in statics is even the same process as the integration they encountered in Calculus, indicating a failure of transfer [51]. Another study examined homework in an introduction to electronics class, where the most important was management of units and orders of magnitude mathematics, but the mathematics in the course was too low-level to compare its content against that of calculus [52]. These previous works, analyzing a single process (i.e., integration) or a first-semester course, lacked the curriculum-wide scope to compare calculus as a whole course against the curriculum of courses that follow it.

Research Questions

While there have been studies that have examined the alignment of curriculum through faculty self-report, these suffer reliability problems. And studies focused on homeworks have insufficiently comprehensive scope to examine questions at the curricular level. Because of the reliability problems with faculty self-report, and scope problems with homework problem analysis, results from previous work may be less persuasive in promoting the curricular change it hoped to promote. So, in the spirit of creating “clear understanding of what is needed”, this study seeks to provide a more reliable, more comprehensively scoped articulation of
engineering’s mathematical needs and expands the previous literature by analyzing the ways that calculus is applied in homework problems in engineering statics.

**Research Question 1)** Which concepts and skills from Calculus are applied in engineering statics homework?

**Research Question 2)** How are the skills that are applied in statics homework used differently than in calculus?

Methods

**Mathematics-in-Use**

The present study examines how prerequisite knowledge from Calculus is applied on engineering homework problems. This approach also preserves the how often calculus content appears in particular courses, which allows for analysis of prerequisite chains.

We employed “mathematics-in-use” technique for the analysis of course artifacts [53]. This technique involves solving a problem completely without skipping a single step, including steps that might be obvious to an expert.
<table>
<thead>
<tr>
<th>Calculus concepts</th>
<th>Calculus skills</th>
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<tbody>
<tr>
<td><strong>Topics</strong></td>
<td><strong>Skills</strong></td>
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<tr>
<td>derivative</td>
<td>definitions &amp; notations</td>
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<td>integral</td>
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<td>fundamental thm</td>
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<td>limit</td>
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<td>approximation</td>
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<td>optimization</td>
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<td>integration techniques</td>
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<td>limit calculations</td>
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<td>sequences/series</td>
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<td>algebraic expressions</td>
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<td>Listening &amp; reading comprehension</td>
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Table 1) An example matrix for the course Differential Equations [53]. Note the calculus concept and skill rows in pink never reach application at any point during Differential Equations.

The resulting data is a narrative description of the problem solution and a list of the concepts and skills that are needed to solve that problem. An example matrix is shown in Table 1. Mathematics-in-use makes an important distinction between topics and skills/concepts [53]. Topics are broad ideas found in tables of contents, syllabi, or at the top of lecture slides (e.g. “Bending Moments”). In this study, the topics are those from Statics. Concepts are lower-level ideas about objects (e.g. such as “derivatives express a rate of change”. Skills are the procedural sequences of steps used to solve a particular type of problem (e.g. “how to compute the derivative of a polynomial”). In this study, the concepts and skills are from calculus, derived from interviews from veteran calculus instructors [54], and the topics are from the course syllabus. The mathematics-in-use technique must document all the ways the student could be expected to know how to solve the problem. For example, a 1st order differential equation can be solved with separation of variables or integrating factor.
Data Selection

We have chosen to study how calculus skills/concepts are applied in the high-enrollment core engineering course Statics. Statics is often the first engineering courses that students take following the calculus sequence and is considered the gateway to upper-level engineering courses. Seven of the 13 engineering degrees at our institution require Statics, which requires Calculus I as a prerequisite. The content of Statics is consistent between institutions [55] so the analysis should generalize well. The median incoming engineering student at our institution has an ACT math of 34 and most students have AP calculus credit. The homework problems and instructor solutions were obtained from the instructor at our institution. The sample has 12 homework assignments with a total of 84 problems.

Results

Based on their prerequisites, one might conclude that students need much knowledge of calculus to succeed in these core engineering courses. However, the volume of calculus knowledge demanded is smaller than one might imagine.
Table 2: Concepts and skills from calculus are on the left axis. Topics in Engineering Statics are along the top. A filled square indicates that topic in the corresponding column applies the concept/skill in the corresponding row. Yellow: Calculus applied. Blue: Calculus possible, instructor solution is algebraic. Grey: Pre-calculus knowledge.

As can be clearly seen in the Table 2, the majority of the mathematics applied in Statics is algebra, geometry, and trigonometry (grey squares in Table 2). Very little calculus is used. Of all the problems in an entire semester of statics (84 problems), just 7 (8%) of these problems require calculus to solve. Five of those 7 are from a single lesson (internal forces). Assuming exam questions had the same distribution as the homework, this would mean that a strong student with no calculus knowledge could get an A- in Statics. In our course, energy functions and unstable equilibrium were not covered, leading to the lack of derivatives and optimization as observed in statics by others [56].
Discussion

Research Question 1

“Which concepts and skills from Calculus are applied in engineering statics?”

The answer to this question is sobering. The skills from Calculus that are applied in Statics are low in both abundance and diversity. Only 8% of problems in statics apply calculus in any way, and the portion of calculus that is applied is very limited. Given the results, it seems likely that the correlations of performance between statics and calculus are not due to direct causation, but rather that both depend on a more fundamental confounding factor: algebra skill.

Recall and Transfer

Many students enter engineering with the belief that mathematics is not truly relevant to engineering, but only a barrier to get through [57, 58]. The tiny quantities of calculus being applied in courses may reinforce this view among students. They may be surprised when more advanced courses like Controls require mathematics that they have dismissed and forgotten.

<table>
<thead>
<tr>
<th></th>
<th>Example Skill/Concept</th>
<th>Calculus I</th>
<th>Statics</th>
<th>Dynamics</th>
<th>IMPLICATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Integration of polynomials</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>Longitudinal reinforcement</td>
</tr>
<tr>
<td>B</td>
<td>Computation of derivatives</td>
<td>x</td>
<td></td>
<td>x</td>
<td>85% Forgotten</td>
</tr>
<tr>
<td>C</td>
<td>$\varepsilon\delta$ limit Quotient Rule</td>
<td>x</td>
<td></td>
<td></td>
<td>Pure math? Obsolete?</td>
</tr>
</tbody>
</table>

Table 4: A mock-up of the consequences of several curricular cases, using the prerequisite chain from Calculus I to Statics to Dynamics as an example. (Dynamics was not analyzed in this study, this table is merely a discussion aid.)

Table 4 shows some possibilities for the curricular path though an engineering sequence. Content in row A is repeatedly refreshed and re-learned through new contexts. The prerequisite system functions as intended, assuming adequate transfer. Content in row B doesn’t occur in Statics, but does occur in the following course, Dynamics. Techniques taught in Calculus that aren’t refreshed may be forgotten by the time they must be recalled in Dynamics [33-38]. Mathematical content in row C may not have application anywhere in the sequence. It may be targeted at
mathematics majors. Alternatively it may be an obsolete technique no longer actively used in engineering [59, 60, 47, 42, 62].

**Research Question 2**

“How are the skills that are applied in statics used differently than in calculus?”

Of the content from calculus that is applied in Statics, the usage and nature of that content knowledge varies considerably. The following section illuminates a few of these differences.

**Complexity of Functions**

In Calculus, students learn a set of rules powerful enough to take the derivative of any combination of functions. Statics coursework does not require students to use any type of derivatives, which means students do not use the chain rule, product rule, or quotient rule at all. Students do use integration, but only integrate piecewise polynomial functions and completely ignore functions such as exponentials, logs and roots. Much class time in Calculus is dedicated to techniques that to not reach application, at least not immediately in Statics [44, 47, 59, 60, 61].

**Continuity**

The continuity concept has interesting epistemic mismatches between calculus and the applied courses. Continuity appears as a necessary concept in 4/7 of the statics problems that use calculus. In Calculus, continuity is just a “property to be checked” [53]. Students are tested on evaluating whether or not a given function is continuous.

In Statics, continuity determines what is allowed. Students must manage which quantities are allowed to be discontinuous, and under what conditions. In Statics, the shear force on a beam can only jump (be discontinuous) at the location of a point load, and the magnitude of that discontinuity must be equal to the size of the point load. Continuity constraints must be applied to solve many problems in statics, particularly with piecewise functions. In Statics, most integration and differentiation acts on piecewise (often piecewise linear) functions. This contrasts with instruction in calculus, where most integrations are of single algebraic expressions. Often in Statics, the explicit algebraic expression for these piecewise functions is not given. An expression must be obtained from a given graph, or the function must be integrated graphically rather than symbolically. Solutions are expected in graph form, not algebraic form. The continuity relationships between segments are always related to physical events or locations, such as the presence of a point load.
Figure 2: Typical function from Statics (left) from the instructor solutions. Piecewise functions containing quadratic or linear segments are the norm.

Reduction to Algebra

Mathematics-In-Use follows all the solution paths that a student might have use. Many problems could be solved using the methods taught in calculus (blue squares in Table 2). However, the instructor solution to these problems does not use calculus and is a purely algebraic solution. Many problems involving bending moment are in principle problems of integration, but this is untrue from the students’ perspective. Most of the “integrations” are over simple rectangular or triangular functions and use basic geometry, not calculus. Students avoid use of calculus when computing bending moments by using the provided table of common centroids.

Limitations

We must interpret these results conservatively.

- This analysis covers only one course at one institution. Other core courses may apply more calculus. Other institutions may have different topic emphasis.
- This study does not measure the quantity of recall nor transfer by students.
- This study only describes Statics as it is, not why it is, or what it should be. Engineering faculty may wish to use more estimation/approximation [62], but feel they cannot, or may have decreased the quantity of calculus in response to falling mathematical competence[18].
- This study cannot answer why a particular course is a prerequisite. It may be a prerequisite for gatekeeping as practice with fundamental skills.
- This analysis does not consider successive re-learning, but successive relearning is unlikely given the 6-8 month time spans between learning and re-learning for this context [63].
• This analysis investigates only homework problems and does not consider other assessments.

Conclusions

From these data, we cannot be too hasty to make changes or suggest reform. However, these results can suggest a shape for future research and discussions. The relatively narrow span of applications may be a boon. Perhaps teachers of calculus who feel they are “in a rush to cover everything” [64-66] could slow down and deepen some content at the expense of less-vital advanced techniques [67, 68].

Specific, detailed mapping of the mathematical needs of the engineering curriculum could allow engineering departments can more specifically articulate their needs to mathematics departments. Mathematics faculty are aware that engineers are dissatisfied with calculus outcomes and want to change to please, but are themselves not well-versed enough in the applications of calculus to do so alone [42, 44]. We can work towards a future where students in mathematics courses need not ask the ever-present question: “When am I ever going to use this?”

Bibliography


