



## **Work-in-Progress: Statistics Components in a Circuits Laboratory for ECE Sophomores**

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## Abstract

Statistics concepts are required for undergraduate curricula in electrical engineering and computer engineering (ECE). Accreditation guidelines specify that such instruction must include “applications appropriate to the program name” [ABET Criteria for Accrediting Engineering Programs, 2014-2015]. Curricula content in probability and statistics are commonly done in a stand-alone course, but additional exposure within engineering-content courses can provide great value. Many basic concepts can be introduced before the comprehensive stand-alone course. A good ECE context is an electrical circuits laboratory in which students need to understand electrical component tolerances and related design issues. Statistics-related assignments in a required, sophomore-level laboratory course are described for applications involving resistor tolerances. Exposure to basic statistics concepts and computer resources in an introductory in-major course facilitates more advanced instruction in follow-on in-major courses.

## Introduction

Statistics are an important part of electrical and computer engineering (ECE) instruction and has explicit requirements within the accreditation guidelines from ABET<sup>1</sup>. Comprehensive courses in probability and statistics are common curricula requirements and these courses may include ECE applications. Early introduction of basic statistical concepts can help prepare students for the stand-alone course and can facilitate more advanced instruction later in the follow-on in-major courses<sup>2,3,4</sup>. Statistics problems within engineering-content courses can provide additional exposure and reinforcement while directly addressing the need for applications in the context of ECE. Much effort has been devoted to integrating statistics throughout students’ undergraduate instruction and to incorporate engineering content<sup>5,6,7,8,9,10</sup>. Laboratory courses are especially useful since data must be generated, manipulated, and interpreted<sup>11,12</sup>. Often, computer tools with statistics capability are being used for other purposes<sup>13</sup>. Finding the right balance among engineering content, statistical concepts, and supporting computer tools is difficult.

This paper investigates the incorporation of statistics-related assignments in a required, sophomore-level laboratory course. Basic statistical concepts are identified which are related directly to data analysis and supported by available computer tools. This circuit analysis laboratory introduces statistics prior to the comprehensive probability and statistics course in the curricula and is a prerequisite for all other in-major courses. The ECE concepts are the measurement and analysis of simple resistor networks and the experiment subject area is resistor tolerance and network simulation<sup>14</sup>. In the first set of exercises, the standard color code and tolerance designations and hands-on resistance measurements provide the data for analysis. Concepts of nominal values, average values, distribution curves, etc. are explored through calculations and plots. The availability of inexpensive resistors with different values and tolerances allow great flexibility in the actual data used, e.g. each laboratory section can have custom data sets. In the second set of exercises, the effect of tolerances on and the statistical

characterization of circuit behavior are demonstrated using common computer tools. These descriptive statistics concepts fit well with the ECE content and the assignments support learning of computer resources for data acquisition, circuit simulation, and computer design tools. The exercises can fit in lecture courses and virtual laboratories if the resistor data sets are provided. The assignments have been developed and are being implemented as a permanent part of the curricula for both electrical and computer engineering majors.

## Probability and Statistics in Electrical and Computer Engineering

### A. Curricula Overview for Probability and Statistics

All undergraduate students pursuing a bachelor’s degree in electrical engineering or a bachelor’s degree in computer engineering at Missouri University of Science and Technology are required to take a probability and statistics course which is taught by the Mathematics and Statistics Department. The required course is “Introduction to Probability and Statistics” (Statistics 3117). The catalog description of the course is<sup>15</sup>:

“Introduction to probability, distribution theory, statistical inference, with applications to physical and engineering sciences. Probability, probability and joint distributions, functions of random variables, system reliability, point and interval estimation, testing hypotheses, regression analysis.” Prerequisite: Calculus III with a grade of “C” or better.

While the course has a good coverage of the statistical topics listed in the catalog description, most of the examples fall into the physical or social sciences rather than engineering. The students learn statistical techniques but they do not see the correlation with their own curriculum. Statistics becomes, in their minds, a separate course with little or no application beyond passing a requirement for their degree<sup>16</sup>.

The degree sequences, especially with regard to the mathematics content, are similar for both majors as shown in Figure 1. The course is recommended to be taken in the junior year when the students are taking their core electrical or computer engineering courses. Many do not take it until their senior year. The course is not a prerequisite for other courses except for the final senior design course.

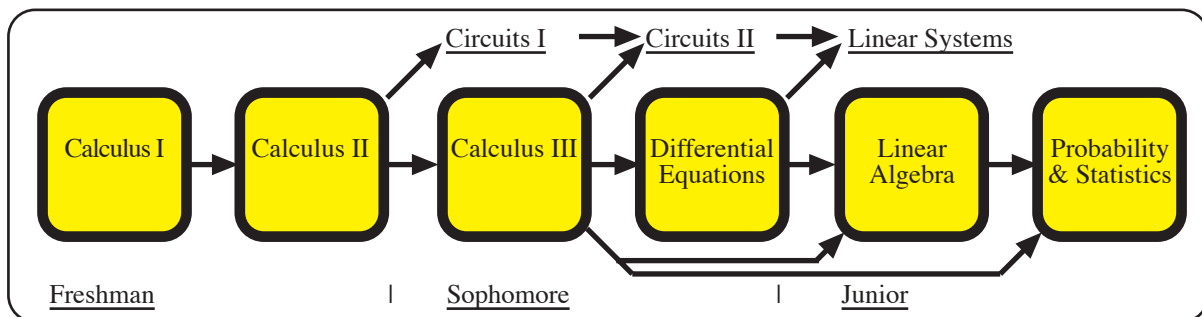


Figure 1. Mathematics Sequence for ECE Undergraduates

The course also is primarily taught as a theory course with paper-and-pencil homework assigned. Students are not aware of the software which is available and commonly used to analyze data. Data sets are small and generally provided to the students without explanation of the requirements of how such data should be collected.

### *B. Curriculum Development*

The motivations for better integrating statistics concepts into the ECE curricula through an early, required laboratory include:

- Providing experience with the application of statistical concepts to ECE problems,
- Strengthening laboratory skills regarding data analysis and manipulation,
- Adding laboratory content that requires use of computer tools,
- Providing additional mechanisms to assess the undergraduate programs (with regard to probability and statistics, experiments, and computer tools), and
- Creating a mindset that statistics is directly related to engineering practice.

As discussed before, the curricula have weak linkages among probability and statistics and the in-major ECE theory and laboratories. For instance, students collect data in laboratory experiments and then they may not know how to apply the techniques they have learned to the data and may fail to see the value of such analysis. By adding a formal introduction to basic statistical concepts in a sophomore in-major course, learning is reinforced along both the major and the mathematics sequences.

This work seeks to introduce the students to basic statistical analysis early on in their program of study, tie the analysis directly to a topic relevant their electrical or computer engineering courses, and to introduce the student to computer aids for data analysis. With this sophomore-level background, further integration may be facilitated through future curricula developments. Such curricula components address various ABET assessment outcomes, e.g. “a) an ability to apply knowledge of mathematic, science, and engineering; b) an ability to design and conduct experiments, as well as to analyze and interpret data; and k) an ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.” Also, the ABET program criteria for ECE explicitly note probability and statistics content<sup>1</sup>.

The circuit analysis laboratory (Electrical Engineering 2101) was chosen as the best location for this in-major introduction to statistics. All electrical and computer engineering undergraduates take the laboratory and the associated lecture course as their initial in-major course, usually when they are sophomores. Students at this level are in mathematics courses which may not allow calculators or other computer aids. To offset this mindset, the laboratory course provides an excellent opportunity to introduce students to another side of mathematics and statistics than the paper-and-pencil calculations they have done thus far in their courses. Computer analysis of data and computer simulation are important parts of any engineering curriculum and an early introduction has great value.

### *C. Computer Tools*

Availability of different tools used for statistical computation or analysis provides enhanced learning for the students. A wide range of tools exists for varied extent of statistical analysis. At the sophomore level, very usable statistical parameters to be understood are measures of center (mean and median) and measures of variability (maximum value, minimum value, range, standard deviation, and quartiles). Usage of computer tools like MS Excel sheets, MATLAB, SAS, etc. offer technology-based time-efficient solutions and learning. In addition, the flexibility of random normalized data set generation offered in these tools is a useful feature. This data variability can ensure independent analysis on the students' part as the data set will not be limited by the components available in the laboratories. Since these computer tools can be used by the students as they progress in their degree programs, the early application of these tools provides another incentive to learn a new tool. Most commonly used computer tools for basic statistical computation are MS Excel, MATLAB, SAS, etc.

#### *MS Excel and MATLAB*

Number sheet formats like MS Excel and Numbers are easy-to-use tools that come with existing functions for basic statistical calculations, e.g. `min()`, `max()`, and `stdev` to find minimum value, maximum value, and standard deviation of a data set. Like Excel, MATLAB also has existing short commands that can be used for basic statistics, e.g. `mean()` to find the average of a data set.

Excel and MATLAB are user-friendly tools and can be used with minimal or no instruction. Both the packages offer great "help" features where an exact function command can be found by keyword. In addition to providing the exact command function to be used, "help" features also provide examples. These types of tools provide better opportunities for self-learning and motivation to students since they are not dependent on class instruction. These types of packages are readily available and reasonably priced making these more attractive to students. On most campuses, these tools are available for free to the students.

#### *SAS and Minitab*

Some of the more specific statistical tools like SAS or Minitab offer more detailed analysis since these tools are developed for statistical analysis only. Advantages of using these tools include the availability of larger data sets, multiple parameters can be tested or evaluated by a single set of commands, and the same or similar tools are used industry wide on a larger scale.

On the other hand, these tools are difficult to learn without instruction and may be time consuming for basic statistics. For computing the minimum value amongst a data set, at least five lines of code are required. Since these tools are syntax specific and there are no prompts or suggestions, students may be more reluctant to use them. These types of packages are highly priced and not easily available. Their licensed installation is usually limited to a very few units on campus and these cannot be accessed remotely thus making them difficult to use on day-to-day basis.

## Statistics Content for Laboratory

### A. Assignment Objectives

The circuit analysis laboratory has the following catalog description.

“Safety, basic measurements and meters, oscilloscopes, resistor networks, measurement of capacitors and inductors, RLC circuit response.” Prerequisite: Preceded or accompanied by Circuits I.

The mathematics prerequisite is Calculus II. The laboratory has ten experiments and the assignments include laboratory notebooks and laboratory reports. Two experiments were selected for revision: Resistive Networks and Analysis of Resistive Values. The original laboratory objectives for the former experiment were to introduce the resistor color code, to verify Kirchhoff’s voltage and current laws through measurements, and to reinforce current and voltage divider relationships. A new objective was added to introduce descriptive statistical concepts in the context of experimental data sets. The original laboratory objectives for the latter experiment were to learn a circuit simulation package. The objective was changed to learn how to perform descriptive statistical analysis using computer tools. These added statistics concepts serve to familiarize students with basic statistics before the required course is taken; to encourage appropriate use of statistics, e.g. error analysis, in future laboratories; and to provide more exposure with statistical computer tools. Since this formal introduction to basic statistics occurs in a course that is prerequisite to all electrical engineering in-major courses, future curriculum revision related to statistics is promoted. Student performance in these experiments will become measures for the ABET assessment plan.

### B. Target Concepts

To assist with the analysis, manipulation, and interpretation of data set, especially in the laboratory environment, the target concepts for the circuit analysis laboratory were descriptive statistics measures. Table 1 and Table 2 list the target concepts and their definitions.

Table 1. Descriptive Statistical Parameters: Measures of Center

Measure	Definition and Calculation
Mean	Arithmetic average of a set of n numbers
	$\bar{x} = \frac{\sum_{i=1}^n x_i}{n} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$
Median	The middle data point in an ordered list of n numbers
	$\bar{x}_{\frac{0}{2}} = \begin{cases} \text{middle value if } n \text{ is odd} \\ \text{average of two middle values if } n \text{ is even} \end{cases}$

Table 2. Descriptive Statistical Parameters: Measures of Variability

Measure	Definition and Calculation
Maximum	$\max()$ is the largest data value in the set
Minimum	$\min()$ is the smallest data value in the set
Range	maximum data value minus minimum data value
Standard Deviation	a measure of the variability (spread) of a data set containing $n$ data points and is based up on the deviation of each data point from the mean
	$s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$
Quartiles Q1 and Q3	Q1=first quartile=lower fourth of the data
	$Q_1 = \begin{cases} \text{median of smallest } n/2 \text{ observations if } n \text{ is even} \\ \text{median of the smallest } (n+1)/2 \text{ observations if } n \text{ is odd} \end{cases}$
	Q3=third quartile=upper fourth of the data
	$Q_3 = \begin{cases} \text{median of largest } n/2 \text{ observations if } n \text{ is even} \\ \text{median of the largest } (n+1)/2 \text{ observations if } n \text{ is odd} \end{cases}$

Data may be plotted in a dot plot or histogram if the data set is small. Alternatively, the data can be graphed in a boxplot as given in Figure 2. In the boxplot, the median and quartiles are shown explicitly on a line segment. Outliers are data values that lie outside the interval  $(Q_1 - 1.5IQR, Q_3 + 1.5IQR)$ .

If they exist, they are plotted as dots.

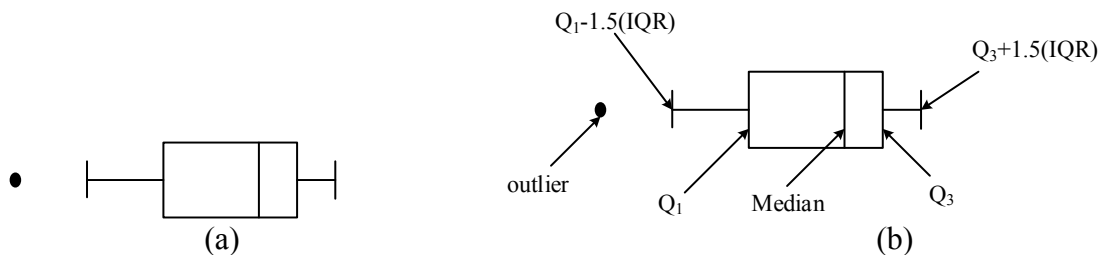


Figure 2. a) Boxplot and b) Boxplot with Key Measures Identified

### C. Laboratory Descriptions and Engineering Application

The laboratory procedure and analysis of experiments for Resistive Networks and Analysis of Resistive Values are shown in Appendix A. (Each experiment includes a background section that is not shown.) The statistical content is incorporated such that instructors can easily vary results among different student teams in the same laboratory section and among different semesters of the course. Many resistor sets can be obtained with the same nominal value, but different distributions within tolerances.

For Resistive Networks, the students are presented with the practical issue of identifying resistors using the conventional resistor four-band color code as given in Table 3. A resistor of nominal value  $4.7 \text{ k}\Omega$  with a tolerance of  $\pm 5\%$  is shown in Figure 3. Actual resistances are measured for a set of resistors and compared to the nominal values and the tolerances as given by the specific resistor labeling. A simple resistor network is constructed and voltage relationships are checked using actual values (again highlighting the difference from nominal values). Then, a set of ten resistors with the same nominal value are selected, measured, and analyzed; the data set is characterized with a dot plot and the associated statistical values. A two-resistor voltage divider is constructed with these resistors and the resistor voltages are investigated for differences from the nominal result of one-half of the source voltage appearing across each resistor. A good introductory design problem is presented in which the allowable resistor tolerance is determined that produces a given plus/minus worst-case variation in the voltage-divider performance.

Table 3. Resistor Four-Band Color Code

<b>First Three Color Bands</b> <b>(In Order: First Significant Digit, Second Significant Digit, and Zeros/Multiplier)</b>					
Band Color	Black	Brown	Red	Orange	Yellow
Value	0	1	2	3	4
Band Color	Green	Blue	Violet	Gray	White
Value	5	6	7	8	9
<b>Last Color Band (Tolerance)</b>					
Band Color	None	Silver	Gold	Red	Brown
Value	20%	10%	5%	2%	1%

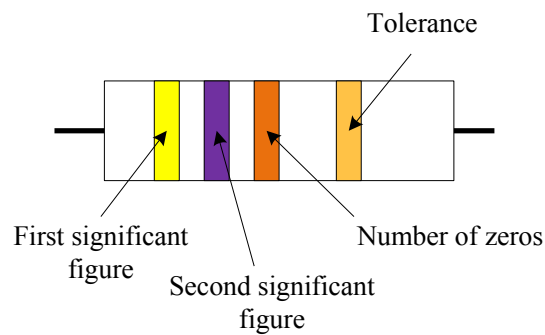


Figure 3. Example Resistor of Nominal Value  $4.7 \text{ k}\Omega \pm 5\%$ .



For Analysis of Resistive Networks, the students are presented with a distribution of 100 resistor values. This data set matches a sample set of resistors with a given nominal value and tolerance. The data set can be computer generated or related to an experimental resistor set. For the initial laboratory procedure, a data set is generated using Excel and a statistical analysis is done. The data manipulation results in a boxplot and a demonstration of various error bar definitions. Table 4 shows selected statistical functions that are built in Excel. Options to this procedure could include comparison of the statistical analysis for different data set distributions and comparison of results from different computer tools.

Table 4. Selected Excel Functions

Function	Command
Random Values	" = <b>RANDBETWEEN()</b> "
Mean	" = <b>average(C1:C100)</b> "
Median	" = <b>median(C1:C100)</b> "
Maximum	" = <b>max(C1:C100)</b> "
Minimum	" = <b>min(C1:C100)</b> "
Standard Deviation	" = <b>stdev()</b> "
Quartiles Q1	" = <b>percentile(c1:c100, 0.25)</b> "
Quartile Q3	" = <b>percentile(c1:c100, 0.75)</b> "

## Preliminary Results

### A. Sample Work for Resistive Networks

Consider a resistor specification as shown in Figure 3, e.g. nominal value 4.7 k $\Omega$  with tolerance  $\pm 5\%$ . Hence, resistance may vary between 4465  $\Omega$  and 4935  $\Omega$ . A corresponding data set is given in Table 5, the descriptive statistics are given in Table 6, and a dot plot is given in Figure 4. Note that the data distribution is bimodal as expected if the  $+2\%$ -tolerance resistors were removed during manufacture.

Table 5. Sample Resistor Data Set for 4.7 k $\Omega$   $\pm 5\%$ . (Ordered)

Ordered Data Set (n = 10) Bands: Yellow – Violet – Orange with Tolerance Red					
Resistances $R_n$	4500 $\Omega$	4555 $\Omega$	4570 $\Omega$	4600 $\Omega$	4795 $\Omega$
	4805 $\Omega$	4835 $\Omega$	4835 $\Omega$	4860 $\Omega$	4925 $\Omega$

Table 6. Descriptive Statistics for Sample Resistor Set

Measure	Value(s)	Measure	Value(s)
Mean	4728 $\Omega$	Maximum	4925 $\Omega$
Median	4800 $\Omega$	Minimum	4500 $\Omega$
Standard Deviation	153.8 $\Omega$	Range	425 $\Omega$

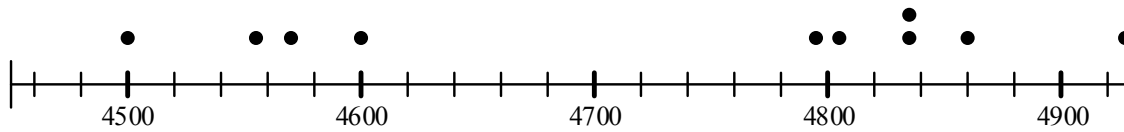


Figure 4. Dot Plot of Sample Resistor Set

The effect of resistor tolerances on circuit network behavior is illustrated by a simple design problem. If the output voltage for a two-resistor voltage divider has a design constraint with respect to its behavior for nominal resistor values, what resistor tolerance must be used? For instance, the design voltage  $V_R$  across either resistor in the divider (“identical” resistors) is one-half of the source voltage  $V_S$  using nominal resistance values  $R_n$ . If  $V_R$  is must be limited to 10% of the design value, then  $V_R/V_S = \frac{1}{2} \pm \Delta V/V_S = \frac{1}{2} (1 \pm 0.1)$  where  $\Delta V/V_S = 0.05$ . Students must observe that the worst-case circuit responses are

$$V_{R,Max}/V_S = (R_n + \Delta R)/(R_n + \Delta R + R_n - \Delta R) = (R_n + \Delta R)/(2R_n) = \frac{1}{2} + \Delta R/2R_n \text{ and}$$

$$V_{R,Min}/V_S = (R_n - \Delta R)/(R_n - \Delta R + R_n + \Delta R) = (R_n - \Delta R)/(2R_n) = \frac{1}{2} - \Delta R/2R_n \text{ or}$$

$$V_R/V_S = \frac{1}{2} \pm \Delta R/2R_n = \frac{1}{2} \pm \Delta V/V_S.$$

The required resistor tolerance is  $\Delta R/R_n = 2\Delta V/V_S$  for a resistor tolerance of  $100 \times \Delta R/R_n = 10\%$ .

### B. Sample Work for Analysis of Resistive Values

Consider a resistor specification for a nominal value of  $1000 \Omega$  with tolerance  $\pm 10\%$ . Hence, resistance may vary between  $900 \Omega$  and  $1100 \Omega$ . A normal distribution is selected. The descriptive statistics are given in Table 7 and a boxplot is given in Figure 5.

Table 7. Descriptive Statistics for Sample Resistor Set

Measure	Value(s)	Measure	Value(s)
Mean	$1008 \Omega$	Maximum	$1100 \Omega$
Median	$1005 \Omega$	Minimum	$904 \Omega$
Standard Deviation	$56.8 \Omega$	Range	$196 \Omega$
Quartile Q1	$971.5 \Omega$	Quartile Q3	$1053.5 \Omega$

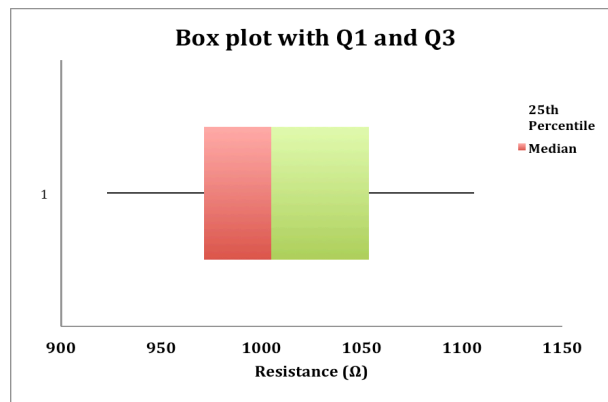


Figure 5. Boxplot of Sample Resistor Set

### *C. Implementation Plan and Discussion*

The assignments have been made a permanent part of the circuit analysis laboratory in ECE at Missouri University of Science and Technology<sup>14</sup>. Student performance, student comments, and instructor observations will be used to modify the assignments for effectiveness. In particular, the laboratory reports have been selected as measures for the department assessment plan regarding ABET outcomes b) and k). For instance, the student grades on the Resistive Networks reports will address the criteria for outcome b) to organize and interpret laboratory data with a goal of having a median score on this selected assignment of 80 percent or more.

This laboratory implementation provides an early basic introduction to descriptive statistics in a more formal way than prior curricula. It is the first step in integrating statistics in subsequent laboratories and in selected lecture courses. As such, the instruction supplements the stand-alone required course in probability and statistics and provides additional learning opportunities involving computer resources for data acquisition, circuit simulation, and design tools. The next laboratory in the ECE sequence is the electronic devices laboratory. The proposed statistic content components here are to extend the influence of resistor tolerances on transistor circuit behavior using MATLAB and to have a statistical analysis in the laboratory design project.

The pedagogical issues that will need to be addressed as the integrated statistics content is further refined in this laboratory and developed for later in-major courses include the following questions. Does the approach

- Provide enhanced learning of statistical concepts,
- Create effective linkage of statistics as a resource for problem solving, and
- Encourage the utilization of computer tools?

### *D. Preliminary Evaluation*

Information from forty-eight undergraduates during the initial semester of implementation was obtained. The assignment grades for the two experiments were good, i.e. average grades of 96.5% and 93.9%, respectively. The evaluation was given as shown in Appendix B. Most of the students were close to the typical degree sequence of required mathematics and physics courses, cf. Figure 1. A majority of the students had completed Calculus II (33 out of 48) and Physics I (42 out of 48). A minority had completed Differential Equations (11 out of 48) and Physics II (19 out of 48). Few students had Statistics (10 out of 48).

The evaluation results are given in Table 8. Note that the subject of resistor tolerances was either a new concept or one that the laboratory experience strengthened understanding for most of the students. Standard deviation was known to most of the students, but the laboratory strengthened understanding for many students. A majority of the students had done laboratory calculations both by hand and by using computer tools, but a significant minority had only used hand calculations. The responses were mixed regarding error bars. The laboratory experiments seemed to be a good introduction to descriptive statistics as applied to an in-major concept, e.g. the responses indicated a weakly positive increase in understanding. As such, the students may be better prepared for similar content in future courses and laboratories.

Table 8. Evaluation Results

<b>Part 1</b>	<b>Pick the Best Choice</b>	
<b>Resistor Tolerance ...</b>		<b>Number</b>
was a new concept.		17
was not new, but I understand how to use the concept in the laboratory better.		24
was a familiar concept.		7
<b>The standard deviation formula ...</b>		<b>Number</b>
was a new concept.		11
was not new, but I understand how to use the concept in the laboratory better.		15
was a familiar concept.		22
<b>Before this laboratory course, I have done laboratory calculations ...</b>		<b>Number</b>
only by hand with pen and paper or a manual calculator.		10
both by hand and by using a computer tools such as Excel, MATLAB, etc.		34
only by using a computer tools such as Excel, MATLAB, etc.		4
<b>Adding error bars to the laboratory plots ...</b>		<b>Number</b>
was helpful in organizing the laboratory results.		13
was somewhat helpful, but I am not sure when it will be needed in the future.		14
was somewhat helpful, but a better description would use maximum and minimum values		8
was not helpful in understanding the laboratory results.		13
<b>Part 2</b>	<b>Rating Scale: Strongly Disagree 1 ... 10 Strongly Agree</b>	
<b>Statement</b>		<b>Average</b>
1. The background and supplementary materials were helpful to understand the concepts.		7.1
2. I have little experience in my other college courses with calculating averages and means.		2.1
3. I have little experience in my other college courses with calculating standard deviations.		5.1
4. I have little experience in my other college courses with data quartiles.		5.5
5. The Excel instructions for performing Experiment 2 were easy to understand.		8.1
6. I had difficulty determining tolerance values from the resistor color code.		3.2
7. I have little experience in my college courses with resistor tolerances.		7.0
8. I understand how different data distributions will change the standard deviation.		6.7
9. I gained a better understanding of experimental error from these experiments.		6.4
10. I feel comfortable adding error bars to my experimental plots in the future.		6.1

## **Summary**

The laboratory assignments illustrate the practical use of resistor tolerances to investigate the performance of circuits and the handling of data. Key aspects of the curricula development have been the more rigorous analysis and interpretation of experimental data, the expanded use of computer tools, and the flexibility of the assignments. For the later aspect, the data sets are easily obtained and they can be continually varied. Each student team and each semester can have a custom data set. The influence of resistor tolerances on circuit behavior can be applied to a wide range of circuits. The preliminary evaluation and the student comments indicate that the laboratory experiments were generally helpful, especially as an introduction to resistor tolerances, and that more background resources or instructions are needed for the laboratory experiments, especially for the use of error bars.

Statistics-related assignments have been incorporated into a required, sophomore-level laboratory course. The laboratory subject is circuit analysis and the assignments investigate the effects of resistor tolerances. This sophomore-level exposure to statistics in the context of ECE applications provides an early introduction to these important statistics concepts and improves the linkage between mathematical analysis and circuit behavior. Follow-on courses in statistics and ECE subject areas can then give more comprehensive coverage and expand the analysis to advanced engineering topics, respectively. Future work is planned in tailoring the assignments and in using the content as prerequisite content for other in-major courses.

## **Acknowledgements**

The support of the ECE Department at Missouri University of Science and Technology is acknowledged.

## Appendix A: Laboratory Experiments with Statistics Content

Laboratory procedure and analysis for Experiment 2: Resistive Networks and Experiment 3: Analysis of Resistive Values in the sophomore-level circuit analysis laboratory<sup>14</sup>. Each experiment includes a background section and other supplementary material that are not shown.

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### Experiment 2: Resistive Networks

#### Procedure

In this experiment, you will construct resistance networks and perform voltage and current measurements to verify some of the basic network relationships you have learned, including Kirchhoff's current law (KCL), Kirchhoff's voltage law (KVL), the current divider relation, and the voltage divider relation. In performing the measurements, keep in mind the internal meter resistance for the DMM determined in the last experiment. Note any instances where this resistance is a significant source of error.

1. Collect 1/4-watt resistors with values of  $R_1 = 4.7 \text{ k}\Omega$ ,  $R_2 = 3.3 \text{ k}\Omega$ ,  $R_3 = 10 \text{ k}\Omega$ ,  $R_4 = 1 \text{ k}\Omega$ , and  $R_5 = 10 \text{ k}\Omega$ . Using the ohmmeter feature of the digital multimeter (DMM), determine the actual values of the resistances. Record the measured resistance values and the tolerance for each resistor. If any of the measured values are not close to the nominal values, double check the resistor labeling and replace the resistor.
2. Using a breadboard, assemble the network shown in Figure 2-1. Using the DMM as a voltmeter, adjust the supply voltage,  $V$ , from the DC power supply, to obtain ten volts.

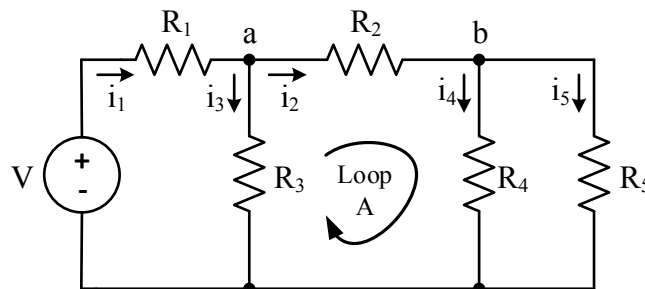


Figure 2-1. Resistor Network with Nodes a and b, note Loop A.

3. Measure and record the voltage across each resistor shown in Figure 2-1. Be sure to record the polarities of the voltages.
4. Using the DMM as an ammeter, measure and record the currents  $i_1$  through  $i_5$  shown in Figure 2-1. You will have to insert the ammeter in series with the resistors one-by-one, and then replace the resistor to its original position prior to doing the next measurement. Keep track of the direction of the currents you measure.
5. Remove resistors  $R_3$  and  $R_5$  from your circuit to create the circuit shown in Figure 2-2.

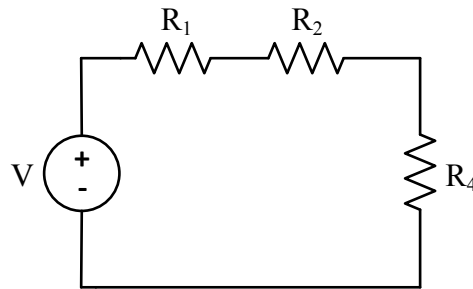


Figure 2-2. Voltage Divider with Three Series Resistors.

6. Measure and record the voltages across the three resistors shown in Figure 2-2. Be sure to record the polarities of the voltages.
7. Collect ten resistors with the same nominal value. Measure the resistance value of each with the LCR meter. Record the measured value and the tolerance (indicated by the color-coded band) of each resistor.
8. Using a breadboard, assemble the network shown in Figure 2-3. Using the DMM as a voltmeter, adjust the supply voltage,  $V$ , from the DC power supply, to obtain ten volts. For  $R_1$  and  $R_2$  use the resistors from step 7 which has the smallest measured value and the largest measured value, respectively. Measure and record the voltages across each of the resistors. Be sure to record the polarities of each of the voltages.

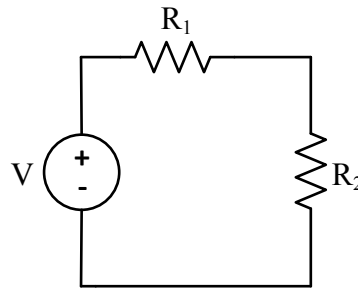


Figure 2-3. Voltage Divider with Two Series Resistors.

### ***Analysis***

In your experiment report, address the following completely through appropriate discussion, data tables, calculations, etc.

1. Verify KVL for the center loop (Loop A) as shown in Figure 2-1.
2. Verify KCL for node **a** as shown in Figure 2-1.
3. Verify the current divider relation for the division of current  $i_2$  as it passes through resistors  $R_4$  and  $R_5$ . You can do this using the currents you measured in step 4.
4. Verify the voltage divider relation for the circuit shown in Figure 2-2.
5. Calculate the percent error from the nominal value for each of the 10 resistors measured.

$$\%error = \frac{Nominal - Measured}{Nominal} \times 100$$

Do any of the resistance values lie outside the allowed tolerance range based up on the color-coded band on the resistor?

6. Make a dot plot for the set of resistor values found in procedure step 7. The endpoints of your line segment should be (nominal value  $\pm$  tolerance). Then plot the values of each of the 10 resistors on the line segment. Do you see a pattern in the data? If so, what might be causing such this pattern?
  7. Calculate the mean (arithmetic average),  $\bar{x}$ , and median of the resistance values for the ten resistors measured in step 7. Are these values close to the nominal value?
  8. Calculate the sample standard deviation,  $s$ , for the resistance values measured in step 7 where  $n$  is the number of resistors in the sample (in this case,  $n=10$ ). Calculate the maximum value, minimum value, and range. How do the maximum and minimum values compare to the endpoints of the dot diagram?
  9. If both resistors were exactly the nominal value, calculate what the voltage would be across each of the resistors in the circuit in Figure 2-3.
  10. Calculate the percent error between the measured voltage value found in step 8 and the “nominal” value found in the previous analysis instruction, i.e. instruction 9.
  11. Calculate the resistor tolerance level required if the output of the voltage divider can vary no more than 5% from the “nominal” value found in instruction 9 of the analysis.
- 

### *Experiment 3: Analysis of Resistive Values*

#### **Procedure**

In this experiment, a randomly generated resistance set will be analyzed to understand the variations in the resistances and other statistically important parameters like mean, median, maximum value, minimum value, range, standard deviation, etc.

1. Use Excel to generate a set of 100 randomly generated resistance values. Your instructor will assign you one of the following resistor values to create the set - 100  $\Omega$ , 330  $\Omega$ , 470  $\Omega$ , 1000  $\Omega$ , 3300  $\Omega$ , or 4700  $\Omega$ . Your instructor will also assign one of the following tolerance values - 1%, 5%, or 10%.  
 Create a new excel sheet and save it as “Last name\_first name\_Circuits1\_Labsection”. Use the command “ = **RANDBETWEEN()**” in a blank cell. In the parenthesis, mention the bottom and top values as calculated for your chosen resistance and tolerance values. For example, if you chose 100  $\Omega$  and a tolerance of 5%, the bottom value will be (100 – 5%)  $\Omega$  and the top value will be (100 + 5%)  $\Omega$ . This should generate one random value. Hold the bottom right corner of the cell in which the value was generated and pull it down the column to generate 100 random values.  
 The following instructions are provided assuming that the data set is located in column A and the data goes from Cell 1 to Cell 100. Select the set of randomly generated number in column A, copy those values and then choose the top cell of column C to paste them. Click on the paste (clipboard) icon and choose “paste special”. Select values and paste the values in the chosen column. This will ensure that you are using the same set of values. Otherwise you may notice that the values keep changing to generate new random numbers.
2. Find the mean of the randomly generated resistances. Choose an empty cell and use the command “ = **average(C1:C100)**” assuming you pasted you values in column C.



Note that you can label your value by choosing a cell right next to the cell you are calculating a value in to label the value, e.g.

999.42	Mean
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3. Find the sample median. Use the command “ = **median(C1:C100)**” in an empty cell. You should see a value that numerically lies right in the middle of the set. You can also use the “sort” icon to sort the data values in ascending or descending order thus verifying the median. Sorting the data will also help you verify or correlate some of the parameters that are to be found in the following tasks.
4. Find the maximum and minimum values amongst the randomly generated set. Use the command “ = **max(C1:C100)**” to display the maximum resistance in the set and the command “ = **min(C1:C100)**” to display the minimum resistance in the set.
5. Find the standard deviation. Use command “ = **stdev( )**” to display the standard deviation of the data.
6. Find the Quartiles Q1 and Q3, i.e. the 25<sup>th</sup> percentile and the 75<sup>th</sup> percentile of the sorted data, respectively. This can be found by using the command “ = **percentile(c1:c100, 0.25)**” and “ = **percentile(c1:c100, 0.75)**”, respectively.
7. Obtain the box plot of the sorted data according to Quartiles Q1 and Q3. Plot Q1, median and Q3 using “clustered stacked bars” under the “charts” tab. By right clicking the chart, choose “format data series”. Then under “options”, select the options to show different colors for the three data points and set the series overlap 100%. Arrange the three stacked bars such that they are overlapping choosing the order from bottom to top as 75<sup>th</sup> percentile, median and 25<sup>th</sup> percentile. Make the 25<sup>th</sup> percentile invisible by making the fill color to white. Add “error bars” to Q1 and Q3 series by choosing “error by percentage”. Error bars feature can be found under the “chart layout” tab. You can choose the percentage to show a smaller or larger range of outliers. Note the other error-bar options.

### *Analysis*

In your experiment report, address the following through appropriate discussion, data tables, calculations, etc. Your Excel file should be attached to the report.

1. Note your assigned nominal resistance and tolerance. Prepare a table showing the following for the data set: mean, median, maximum value, minimum value, range, standard deviation, etc.
2. From Step 7, show the boxplot. Prepare a table showing the median and the Q1 and Q3 quartile values.
3. Error bars could be used to display the expected variation of values around the mean. Error-bar designation could be defined using the maximum and minimum values, the standard deviation, the quartile values, or a given percentage. Describe how many outliers occur in each of these four definitions. Discuss how the number of outliers would change if the data distribution was not uniform across the range, but followed a normal distribution (also called a bell curve or Guassian distribution).

## Appendix B: Evaluation of Statistics Laboratory Experiments

Evaluation form for students that completed Experiment 2: Resistive Networks and Experiment 3: Analysis of Resistive Values in the sophomore-level circuit analysis laboratory.

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### Part 0: General Information:

*What mathematics and physics courses have you completed?* (Select all that apply.)

Calculus III                       Differential Equations                       Linear Algebra  
 Statistics                               Physics I                                       Physics II

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### Part 1: Pick the best choice for each statement.

*Resistor tolerance ...*

- was a new concept.  
 was not new, but I understand how to use the concept in the laboratory better.  
 was a familiar concept.

*The standard deviation formula ...*

- was a new concept.  
 was not new, but I understand how to use the concept in the laboratory better.  
 was a familiar concept.

*Before this laboratory course, I have done laboratory calculations ...*

- only by hand with pen and paper or a manual calculator.  
 both by hand and by using a computer tool such as Excel, MATLAB, etc..  
 only by using a computer tool such as Excel, MATLAB, etc.

*Adding error bars to the laboratory plots ...*

- was helpful in organizing the laboratory results.  
 was somewhat helpful, but I am not sure when it will be needed in the future.  
 was somewhat helpful, but a better description would use maximum and minimum values.  
 was not helpful in understanding the laboratory results.

### Part 2: Please use the following scale to respond to each of the statements in Part 2:

**Strongly Disagree** 1 ... 2 ... 3 ... 4 ... 5 ... 6 ... 7 ... 8 ... 9 ... 10 **Strongly Agree**

1. The background and supplementary materials were helpful to understand the concepts  
 2. I have little experience in my other college courses with calculating averages and means.  
 3. I have little experience in my other college courses with calculating standard deviations.  
 4. I have little experience in my other college courses with data quartiles.  
 5. The Excel instructions for performing Experiment 2 were easy to understand.  
 6. I had difficulty determining tolerance values from the resistor color code.  
 7. I have little experience in my college courses with resistor tolerances.  
 8. I understand how different data distributions will change the standard deviation.  
 9. I gained a better understanding of experimental error from these experiments.  
 10. I feel comfortable adding error bars to my experimental plots in the future.
-

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