A Student-developed Rotational Mechanics Laboratory Exercise to Link Engineering Design and Science

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Introduction

A noted challenge in our curriculum when teaching engineering design or engineering science is a reluctance by students to apply topics learned in their engineering science courses in their engineering design courses and vice versa. This paper reports on the first step—design and development of a rotational mechanics laboratory exercise—toward creating shared experiences that scaffold student learning through deliberate and topically-meaningful activities shared between engineering design and engineering science courses typically completed by sophomore engineering students in our program.

In order to provide context for the rotational mechanics laboratory exercise, we offer the following brief descriptions of the university, program, and courses that the exercise targets.

James Madison University

James Madison University (JMU) is a public regional university located in Harrisonburg, Virginia, within the Shenandoah Valley. James Madison University has a total enrollment of approximately 20,000 students across all of its seven colleges with approximately 1,700 of those students enrolled in a graduate program. The College of Integrated Science and Engineering (CISE) was established in 2012 with a college restructuring and consists of three academic departments: Integrated Science and Technology, Computer Science, and Engineering.

Madison Engineering

The School of Engineering at JMU was founded in 2005 with the first cohort of students starting in fall of 2008. The School of Engineering now exists as the Department of Engineering (or colloquially, as Madison Engineering).

Madison Engineering was designed to be a progressive program unrestricted by the boundaries of traditional engineering disciplines. The program was proposed based on the following description of the Engineer of 2020 by the National Academy of Engineering: one who possesses strong analytical skills, strong communication skills, a strong sense of professionalism, creativity, and versatility. The program is ABET accredited (10/01/2011 – present) under the Engineering Accreditation Commission. The program is not discipline specific and has a current enrollment of approximately 450 students as of August 2014. The program is comprised of 126 credit hours with most students completing the degree in four years.

The Madison Engineering program was established to train the engineering versatilist. Engineering versatilist is a phrase invented by Garner, Inc. and popularized by Friedman that described an individual who can “apply depth of skill to a progressively widening scope of situations and experiences, gaining new competences, building relationships, and assuming new roles.” The program includes a design focus where design is a thread that runs through the curriculum at all academic levels.
Courses Affected by Rotational Mechanics Laboratory Exercise

During the spring semester of their sophomore year, Madison Engineering students typically take a statics and dynamics combined course and a design course (Engineering Design 2). Statics & Dynamics introduces applied mechanics from an engineering standpoint and is the first of five required "engineering science" courses in the curriculum. The statics and dynamics course has three class meetings each week and one lab meeting each week. Course labs have been designed to pair with course classroom content and involve activities such as learning to take force measurements with load cells. Engineering Design 2 is the second design course in a two course sequence (Engineering Design 1 and Engineering Design 2) and introduces students to process-based design in preparation for their capstone sequence. For the past five years and for the foreseeable future, Engineering Design 1 and 2 have focused on the development of a human-powered vehicle for a client. Each year’s client has had a physical condition that makes operation of a traditional bicycle difficult or impossible. In the first semester of the design course sequence, students are grouped in teams to determine the client's needs and formulate designs to meet the needs. In the second semester, the student teams work to assess their conceptual designs and to construct physical prototypes. This design and construction activity provides the students with a yearlong, immersive experience with a client and a mechanical apparatus.

Motivation & Background

Engineering science courses typically include applications and examples that are taken from the "real world". For example, typical problems found in statics and dynamics textbooks introduce students to abstractions for and analysis of devices in common usage including cranes, helicopters, conveyors, and bolt cutters. White found, though, that students, when solving basic physics problems, often draw from “diverse components of their knowledge, which seem relevant to the solution of force and motion problems, but which often have properties that conflict with the implications of Newton’s laws of motion.” Caramazza et al. found similar with a study of undergraduate physics students at Johns Hopkins University, and many similar examples can be found in literature. These findings indicate that students’ mental models (where mental model is defined as “a representation that is isomorphic to the physical situation that it represents and the inference processes the physical processes being reasoned about” of such devices may tend to be poorly developed, lack important details, or be completely incorrect, so students are less likely to internalize and take ownership of their analyses. This incorrect or poorly developed mental model may be referred to as a misconception, and dislodging these misconceptions can be particularly difficult.

Providing contextually-relevant, active learning examples is hypothesized to promote connections to critical details and enhanced student learning in both Statics & Dynamics and Engineering Design 2. This hypothesis is, at least in part, supported by a study performed by Hake comparing pre-/post- assessment results from 62 physical courses representing over 6000 students. Hake found that students who completed courses that made use of “interactive engagement” demonstrated nearly two-standard-deviation gains in conceptual understanding of physics topics when compared to students completing “traditional” courses. And more recently, Freeman et al., published results of a meta-analysis of 225 studies indicating that active learning
has the potential to increase student performance on concept inventories and increase exam scores when compared to traditional lecture courses.  

Specifically, this paper addresses three issues that have been observed by faculty teaching the first two design courses and the statics and dynamics course, and a strategy they used to try to correct these issues. The three issues are (1) a lack of integration between engineering design and engineering science, (2) student misconceptions resulting in erroneous reasoning to explain observed phenomena in engineering design and engineering science, and (3) a reluctance of students to apply their engineering science knowledge when following the engineering design process. Newstetter summarizes the problem as “the scaling up that is required in a number of areas from doing individual back of the book analysis problems alone on paper to building a complex engineered mechanism with several subsystems on a team with four or five other people using suite of tools is too great,” and provides the following advice, “to become competent in each one of these areas—application and integration, team work, and tool use—students need time, repeated experiences, and a lot of reflection on the learning.” The goal of this research project is to create active-learning activities that create meaningful connections between engineering science and engineering design that teach students to apply and integrate when ‘doing design’. This goal is well summarized by Dym as there must be, “a change in attitude toward a more explicit and visible role for design as being ‘what engineering is all about.’ Analysis unquestionably retains its centrality for formulating and modeling engineering problems, and for evaluating design results. The point is that students have to learn engineering science so that they can do design, that is, engineering science is taught to enable our students to be able to do design.”

Toward achieving this objective, for the past three years, students completing Engineering Design 2 have been introduced to mass analysis, frame analysis, and gearing analysis during the preliminary and detailed design phases of the design process. Students complete examples from their Statics & Dynamics course text while in Engineering Design 2, and then apply the analysis to the bike frame designs generated through the Engineering Design 1 and 2 course sequence. This process proves quite challenging for many of the students, but typically, there are a few students who engage in the process and try to understand how to move from calculated forces to stress and strain.

This paper reports on the first attempt at deliberately integrating Engineering Design 2 related topics into the Statics & Dynamics course. This was achieved through student-lead development of a laboratory exercise that could better help students make connections between theory, application, and design. While there is a lot of literature discussing students being tasked with developing their own laboratory assignments, and then completing these assignments themselves, our scenario is more unique in that the student developing and completing the laboratory assignment did so after already completing the class, and developed the assignment with the intent of having other students complete and learn from the assignment. One school that had students develop assignments like this is the University of Wyoming. The engineering department at the University of Wyoming uses an approach they call For Students By Students, and students develop both laboratory exercises and robotic systems that are used to teach other students in their program. These projects are reported to be popular among the students; they help the students acquire skills such as prototyping to meet requirements and interacting with
their clients (the other students) by acting as the TA during the laboratory experiments. It was
stated that in order for a student developed assignment to be successful, the student must be
willing to take on the task and must have the ability to meet the task’s technical demands.

Methodology

Toward the goal, the faculty-student team choose to focus the project on the development of a
rotational mechanics laboratory assignment to be housed in the statics and dynamics course. To
help make the connection between Statics & Dynamics and Engineering Design 2, the student
was told to focus on bicycle wheels, a physical artifact that students taking statics and dynamics
had already physically experienced during their first semester of design during a bicycle tear
down and reconstruction activity. Also, the bicycle wheel is a system noted in literature as one
that frequently challenges students due misconceptions.

A key strategy employed when considering the connections between the courses was to leverage
complementary learning objectives from the two courses during assignment development. The
following learning objectives were identified when proposing the lab development.

*Statics and Dynamics:*
  - Analyze particles, systems of particles, and rigid bodies by applying kinematics, energy, and momentum methods.
  - Conduct an experiment and analyze resulting data.
  - Use computer tools to solve statics and dynamics problems.

*Engineering Design 2:*
  - Construct and assess designs using elementary prototypes.
  - Perform system-level and detailed product design phases.

These five objectives were not given to the student explicitly; however, discussions at the outset
of the project implicitly addressed all five objectives as being key areas to cover during lab
assignment development.

The student had been introduced to the concepts required to successfully develop the lab through
previous courses. Statics and Dynamics introduced the student to concepts of kinematics of
rotation and moment of inertia, provided experience analyzing experimental data in a spreadsheet, and
provided experience using a spreadsheet to develop numerical solutions to dynamics problems. Engineering Design 2 introduced the student to computer-aided design, prototyping, and bicycle mechanics. Although it was anticipated that the above concepts would be reinforced during lab development, the primary learning outcome was to provide experience synthesizing previously learned concepts. This goal was supported by the open-ended nature of the project.

The selected student engaged quickly in the task of developing the laboratory assignment. The
student who was chosen was only one month removed from completion of the statics and
dynamics course and one semester plus one month removed from completion of the sophomore
design sequence. This way, the student’s experience of the course was still fresh in the student’s
mind, and the student would be able to remember difficulties the student had or the student’s peers had making connections while taking the course. The student developing the laboratory exercise had not been exposed to a rotational mechanics laboratory exercise when completing statics and dynamics the previous semester; therefore, the laboratory exercise was developed without any bias toward what a laboratory exercise focused on rotational mechanics should or should not be.

The student was given a budget and was told to use a bicycle wheel and an accelerometer as the main features of the exercise, but was given the freedom to purchase or develop any other laboratory equipment within a $500 budget seen as beneficial to the laboratory exercise created. The student was given the full Spring 2014 semester to develop the rotational mechanics laboratory assignment as well as all of the equipment required to develop the assignment, the lab assignment write-up, and a sample lab report representing what a student assigned the laboratory assignment might ideally turn in as their report. The lab assignment write-up and the sample lab report are provided as Appendices 1 and 2, respectively.

Coincidentally, it should be noted that the student was also enrolled in two technical electives, Introduction to Sensors and Solid Modeling & Prototyping. Through the student’s course work in Introduction to Sensors, the student was able to gain an understanding of how to work with the accelerometer, and through the student’s course work in the Solid Modeling and Prototyping course, the student learned to use CAD software to design and prototype (using the Department’s 3D printers) the laboratory equipment for the assignment. Also, the student works part time as an apprentice machinist in the University’s machine shop; leveraging downtime in the machine shop, the student was able to machine all required metal components required for the lab assignment.

For the basic setup of the laboratory assignment (provided as Figure 1 - left), the student chose to have the bicycle wheel attached to a standard bicycle fork with a quick release on the wheel for easy disassembly. The student designed and constructed a mount to attach the bicycle fork to the tables in the Statics & Dynamics lab. The mount allows the bicycle fork to rotate 360-degrees; a pin keeps the bicycle fork and wheel in position (horizontal or vertical) during lab exercises. Figure 1 shows the configuration for which the bike wheel is oriented parallel to the ground. The student went through multiple designs that could be used to mount the wheel to a table, including a free standing wooden frame. The student found that the wooden frame wobbled when the wheel was spun, producing significant noise in the accelerometer data. The student made their final mount out of aluminum at JMU’s machine shop.
Since the accelerometers are used in more than one lab, attachment of the accelerometer to the bicycle wheel required the student to design an apparatus that allowed for straightforward disassembly. The student designed a 3D printed clip design that attaches to spokes of the wheel. As with the mount, the student went through multiple accelerometer clip designs before developing a functioning clip that would offset the angle of the bicycle spokes. Through experimentation, the student found that the angle of the bicycle spokes caused the accelerometer to measure some gravitational acceleration in the same axis as the measured acceleration caused by the spinning wheel. The student shaped the clip in a way that would allow the accelerometer to be parallel to the ground instead of at the spoke angle; the result is minimized measurement of gravity in the axis of interest.

The student wished for the assignment to require using data from the accelerometer (which takes three separate acceleration measurements; one for each of its three axis) to find angular acceleration. When the wheel was oriented vertically, gravitational acceleration was measured in the same axis as angular acceleration. The student decided that separating gravitational acceleration from angular acceleration through analysis was outside of the scope of the assignment, so the assignment developed uses a horizontal wheel, as illustrated diagrammatically in Figure 1 - right. An accelerometer on a spinning horizontal wheel will measure gravitational acceleration on a different axis than it will measure its acceleration from spinning, so that analysis is not required to separate the two effects.

After developing the setup of the experiment and developing functioning equipment, the student collected data from the accelerometer clipped onto a spinning wheel. After analyzing the data, the student created an assignment that the student thought would be challenging, but also, would signify a students’ understanding of rotational mechanics if completed correctly.

Results

The student developing the exercise produced a laboratory assignment focused on helping students learn rotational mechanics and helping students build connections between engineering
design and engineering science. They produced a sample of a completed lab report, representing what a completed and thorough lab report produced by a team of students should look like. The laboratory assignment is provided as Appendix 1, and the sample completed lab report is provided as Appendix 2. Among other things, the assignment asks students to use data from an accelerometer mounted to a spinning bicycle wheel to find angular acceleration, angular velocity, and angular displacement of the wheel. It also asks students to approximate the force used to spin the wheel, and asks them to approximate how much force it would have taken had they spun it near the center of the wheel instead of spinning it from the wheel’s outer diameter.

It should be noted that the student did not include goals and objectives in the lab assignment. These had to be added following the students completion of the assignment.

In addition to the written assignments and the sample report, the student also developed a sensor mount to attach the accelerometer to the bicycle wheel and a fork mount for securing a bicycle fork to a table. Figure 1 shows the sensor mount and the fork mount that the student developed.

It was clear through discussions with the student related to data collection and analysis that dynamics concepts had been reinforced. For instance, multiple iterations of hardware used to clamp the accelerometer to the wheel spokes were the result of increased familiarity with the hardware used in the lab, in particular, the accelerometer’s frame of reference. Furthermore, the student developing the lab eventually realized that the first version of the lab developed was too difficult for students in the Statics and Dynamics course; this supports the notion that the student experienced a reinforcement of dynamics concepts during development efforts and supports a finding reported by University of Wyoming. Another way to look at this is from the perspective of a second student who opted to complete the lab as a course honors option to provide feedback for continuous improvement efforts. As this second student became familiar with the expectations of the lab, the student decided to not complete the exercise.

The lab assignment that was developed provides students with an opportunity to formally observe familiar physical phenomena (the slowing down of a spinning bicycle wheel), and to quantify the phenomena using concepts from engineering science. As such, it reinforces synergistic connections between students informal observations while developing a human powered vehicle in Engineering Design 2, and concepts learned in Statics and Dynamics. Furthermore, while targeted course outcomes were not explicitly given to the student, the lab exercise developed by the student will serve to articulate three targeted course objectives from Statics and Dynamics, namely:

- Analyze particles, systems of particles, and rigid bodies by applying kinematics, energy, and momentum methods;
- Conduct an experiment and analyze resulting data; and
- Use computer tools to solve statics and dynamics problems.

In Engineering Design 2, though, both desired course objective connections were not met. While the student experienced the process of creating and assessing prototypes, other students completing the lab exercise will not have this same experience, as the lab apparatus has been developed for them. The exercise, will though, serve to demonstrate how analysis being
performed during Statics and Dynamics may be used during detailed design phases, and discussion to this effect will be added to the Engineering Design 2 course schedule.

**Discussion & Future Work**

The development of the Rotational Mechanics Lab activity provided one engineering student a design/build activity that coupled not only the student’s prior engineering science and engineering design course work, but also, the student’s course work from an Introduction to Sensors course and from the Solid Modeling & Prototyping course. The student learning through this process, while perhaps not directly measurable, is evident in the iterative design, prototype, learn loop completed by the student and described in the methodology section. When developing laboratory equipment, the student had to both use an iterative design process, and had to draw on knowledge acquired from taking Statics and Dynamics, an Introduction to Sensors course, a 3D Modeling and Prototyping course, and from working in the University’s machine shop. The student went through three designs before machining a mount that could attach a bicycle fork to a table and estimated going through ten designs before creating a 3D-printed accelerometer clip that attaches the accelerometer to bicycle spokes securely in the proper orientation.

The accelerometer clip especially demonstrated iterative design; since it was 3D-printed, the student was able to redesign, print, test, and evaluate the piece quickly. Some of the problems the student encountered, and had to incorporate into the design, included needing all of the piece’s features to be printable on the school’s 3D printer, needing a clip that could be clipped at different radial distances on the wheel (an issue because distance between spokes changes with radial distance), and needing the clip to orient the accelerometer parallel to the ground so that the accelerometer would only measure gravity on one of its axes (an issue because bike spokes are at an angle).

What the student found unique about this experience is that each of these issues encountered required the student to draw upon multiple knowledge bases to identify solutions. In the student’s classes, the student feels that many times the student only solves problems associated with the particular class they are in. The student was unaccustomed to using knowledge of modeling, 3D printing, and machining to solve design problems. Over the summer, when the student had an engineering internship based in mechanism design and development, the student found himself having to draw upon the same knowledge bases and having to once again use an iterative design approach. The student found that the processes gone through while developing the laboratory equipment closely paralleled and prepared the student for the processes experienced in the work place.

Consequently, we believe that projects and opportunities such as the one outlined herein provide valuable learning opportunities for motivated students. The next step is to understand how the student’s peers will react to the lab activity. The first deployment of the rotational mechanics laboratory exercise will be this upcoming Spring 2015 in one section of Statics and Dynamics with six students. This cohort of Statics and Dynamics students, who are also enrolled in Engineering Design 2, will be the first students to complete the lab activity. Students will work on the lab activity with partners and will perform the lab activity during designated time periods.
as we do not plan to manufacture additional apparatuses until after the lab is tested in a classroom environment.

Assessment will be comprised of three elements. First, students will be assessed using a concept inventory style questionnaire with instrument deployment prior to the use of the lab and then again following the lab. The concept inventory style questionnaire will allow the course instructor to determine prior misconceptions that might exist—perhaps as a result of working on the sophomore bike design project—and whether or not those misconceptions change as a result of working with the Rotational Mechanics Lab. (A Dynamics Concept Inventory has been developed to assess students’ conceptual understanding of dynamics concepts with emphasis on common misconceptions using the Delphi Method, and will be explored for its applicability.28) Second, students will complete a knowledge quiz prior to completing the lab activity to see if students are able to transfer the analysis procedure learned with the Rotational Mechanics Lab to similar problems. Third, students will be asked to reflect on the Rotational Mechanics Lab with respect to prior labs in the end-of-year assessment. It is our plan to not inform the students that the Rotational Mechanics Lab was developed by one of the their peers.

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Appendix 1: Laboratory Exercise

Herein is the laboratory exercise developed by the student. It is anticipated that students would be given the exercise to read prior in advance of showing up to the lab class period.

ENGR 212 – STATICS AND DYNAMICS
Rotational Motion Laboratory Exercise

Introduction

In this laboratory, you will use an accelerometer to gather data from a rotating bicycle wheel. Using data gathered from the accelerometer, you will analyze its rotational motion using concepts we have discussed in class.

Goals & Objectives

- Provide students with hands-on experience with a system in which moment of inertia plays a significant role in system behavior
- Provide students with a more formal construct to address familiar phenomena
- Demonstrate that modeling of collected data can lead to knowledge of physical parameters that are not readily directly measured

Equipment

The settings on your accelerometer need to be set the same way they were in the Pre Lab assignment. Make sure you remember how to properly begin and end data collection when using the accelerometer.

It is important to know the diameter and mass of the wheel you are using. It is also extremely important to measure the distance between the sensing element of the accelerometer and the center point of the wheel.

Horizontal Trial

Orient the wheel so that the wheel’s diameter is parallel to the ground. An example of this orientation can be seen in Figure 1.

Begin collecting data with the accelerometer. Once data collection has begun, wait about ten seconds, and then spin the wheel. Spin the wheel by applying a force to its outer diameter, DO NOT spin it from its spokes or any other part of the wheel. Allow the wheel to spin until it comes to a complete stop on its own. Wait about ten seconds after the wheel comes to a stop, and then end data collection.

It is necessary that you record what direction the wheel and accelerometer were spun so that you can properly interpret the data recorded by the accelerometer. It might be helpful to take a video
of the wheel and accelerometer spinning. You will likely want to redo this horizontal trial a few times so that you can pick the cleanest set of data for your analysis. Note that accelerometer data will need to be converted to $m/s^2$ for the analysis.

![Figure 1: Wheel Positioned for Horizontal Trial](image)

**Vertical Trial**

Orient the wheel so that the wheel’s diameter is perpendicular to the ground. An example of this orientation can be seen in Figure 2.

![Figure 2: Wheel Positioned for Vertical Trial](image)

Repeat the same process that you preformed for the horizontal trial. Once again, it is important to record the direction the wheel and accelerometer were spun.
Lab Report

Each lab group will complete one lab report. The report must contain the following information:

From the Horizontal Trial

1. Plot the acceleration measured in each of the accelerometers three axes on the same graph. Explain what each axis represents using appropriate terms (normal acceleration, tangential acceleration, gravity, etc.).

2. Develop and explain a method to remove gravitational acceleration from the data in order to obtain data that represents the rotational motion of the wheel. Carry out your method, and plot the resulting acceleration in each of the accelerometer’s three axes on the same graph as functions of time.

3. Create a plot of the accelerometer’s angular velocity with respect to time. Include a trend line equation. Explain how you developed the plot, and explain what the trend line equation represents. If the accelerometer were moved closer to the center of the bicycle, would the angular velocity change? Why or why not?

4. Create a plot of the velocity (NOT angular velocity) of an arbitrary point on the tire’s outer diameter. Include a trend line equation. Explain how you developed the plot, and explain what the trend line equation represents.

5. Using one of your trend line equations, develop and discuss equations that can be used to model the wheel’s angular acceleration, angular velocity, and angular displacement. Then find the total distance that the wheel would have traveled during data collection if it had been rolling along the ground instead of spinning in the air.

6. Assume that the accelerometer and its clip have a combined mass of 0.1kg, and assume that all of the wheel’s mass is located at a radial distance one inch inward from its outer diameter. Create a free body diagram that includes the wheel, the axel, the wheel’s mass, the accelerometer, and the force used to spin the wheel.
   i) Find the moment of inertia about the wheel’s axel
   ii) Find the torque and the force that you applied to the wheel’s outer diameter when you spun it, assuming you exerted force on the tire for a duration of one second.
   iii) If you had spun the tire to the same velocity by exerting a force on the tire’s spokes 1 inch from the tire’s axel instead of exerting the force on its outer diameter, would it require more or less torque and force to spin the wheel compared to the values you found in ii? Explain.

7. If the accelerometer and its clip were to be moved closer to the wheel’s axel, and the same torque was applied as in item 6, qualitatively discuss what would happen to the wheel’s:
   i) Moment of inertia
   ii) Maximum angular velocity
   iii) Time it takes for the spinning wheel to come to a complete stop

From the Vertical Trial

8. Plot the data from the accelerometer’s there axes on the same graph, and discuss what each of the accelerometer’s axes are measuring using appropriate terms. Why are there are large oscillations present in this acceleration data?

9. Develop and explain a method for removing the effects of gravity from the accelerometer data. If you cannot develop a method, explain why you cannot develop one and explain what you would need to do to develop one.

The report must be completely typed; nothing should be hand drawn or hand written.
Appendix 2: Sample Lab Report

Herein is a sample lab report written by the student demonstrating an anticipated finished deliverable to be produced by his peers. It is anticipated that a student would complete this report during the week following performing the lab activity, and that this report would be turned on one week after the lab class period.

**Rotational Motion Laboratory Exercise**

5/16/14

**Executive Summary**

In this lab, a bicycle wheel was spun at two different orientations. It was first spun so that its diameter was horizontal to the ground, and it was then spun so that its diameter was perpendicular to the ground. The bicycle wheel had a 22 inch diameter, and had a mass of 1.227 kg. The bicycle wheel was equipped with an accelerometer used for data collection. The accelerometer was located 8 inches from the wheel’s center.

The wheel was first spun horizontally. While the wheel was spinning, the accelerometer collected acceleration data for the accelerometer’s X, Y, and Z axis, which the accelerometer labels Ax, Ay, and Az. The data collected by the accelerometer was recorded a unit called counts. In order to convert counts to $m/s^2$ Equation 1 must be used.

\[ A = \frac{A_{\text{counts}}}{1024} \cdot 9.81 \ m/s^2 \]  

(1)

In Equation 1, A is acceleration with units of $m/s^2$, $A_{\text{counts}}$ is the number of counts measured by the accelerometer, 1024 is a conversion factor that is specific to the accelerometer’s high gain 12-bit resolution setting, and 9.81 $m/s^2$ is acceleration due to gravity, which is needed to convert from g force to $m/s^2$.

Figure 1 shows the acceleration in $m/s^2$ that the accelerometer recorded in each of its three axes when the wheel was spun horizontally. According to a diagram in the accelerometer’s datasheet that displays the accelerometer’s coordinate axis system, Ax points towards the center of the wheel, meaning that Ax represents normal acceleration. Ay points in the direction of the accelerometer’s motion, meaning that Ay represents tangential acceleration. Az points towards the ground, meaning that Az represents acceleration due to gravity.
The data displayed in Figure 1 cannot yet be used to analyze the motion of the wheel because it includes gravitational acceleration along with rotational acceleration. In order to separate the acceleration due to gravity from the acceleration caused by the wheel’s motion, the acceleration measurements taken from t=25s to t=40s can be used, because during this time interval the wheel was not moving. By averaging the acceleration measurements taken by the accelerometer in each of its axes during this time interval, the acceleration due to gravity in each of its axes can be estimated. This average acceleration due to gravity in each of its axes can then be subtracted from each measurement taken by the accelerometer to obtain an approximation for acceleration caused by the wheel’s motion.

The reasoning behind this method is that since the wheel is not moving during this 15 second time interval, the accelerometer is only measuring acceleration due to gravity. Assuming the wheel is level to the ground, gravity should be split between the axes the same way regardless of the accelerometer’s angular position on the wheel. The acceleration measurements during this 15 seconds time interval were averaged together in order to reduce the effects of noise. The three separate average gravitational accelerations were then subtracted from their respective axes at every time interval in order to remove gravitational acceleration and leave only acceleration caused by rotational motion. The results of this method of removing gravity from the accelerometer data are shown in Figure 2.

![Figure 1: Plot of Acceleration Measured by theAccelerometer as a Function of Time](image-url)
Figure 2 shows the acceleration measured by the accelerometer in each of its axes after removing gravity using the described method. By comparing Figure 1 with Figure 2, it can be seen that removing gravity had a very large impact on the acceleration measured on the z axis, because almost all of Az in Figure 1 is caused by gravity. Also, Figure 2 shows no significant acceleration occurring after the wheel came to a stop shortly after the 20 second mark, meaning that there is only acceleration when the wheel is moving. This is evidence that the method used for removing gravity to obtain acceleration data caused only by the wheel’s motion was valid.

The method used for extracting gravity may have been inaccurate if the wheel was tilted. The accelerometer remained at the same orientation from t=25s to t=40s. If the wheel was tilted, acceleration due to gravity would be split between the three axes differently depending on the angular position of the accelerometer. For example, if the wheel was tilted so that an arbitrary point A on the wheel’s outer diameter was higher than any other point on the wheels outer diameter, acceleration due to gravity measured in the accelerometer’s Y-axis would be zero at point A and at the point 180° from point A, but the accelerometer would measure acceleration in the Y-axis at all other angular positions, with maximum magnitudes of acceleration due to gravity in the Y-axis occurring at positions 90°and 270° from point A.

If the wheel was not tilted, there should be no acceleration detected in the accelerometer’s y-axis then the wheel is at rest. There was a small amount of acceleration measured in the y-axis when the wheel was at rest (about 0.14 m/s²), so either the wheel or the accelerometer must have been tilted, meaning the method used for extracting gravity was not perfect. The oscillations that can be seen in the acceleration plots in Figure 1 and Figure 2 also indicate that the wheel or accelerometer were likely tilted. However, because 0.14 m/s² is small relative to total acceleration due to gravity, 9.81m/s², it can be assumed that this method for extracting gravity from accelerometer data worked well enough to analyze the wheel’s rotational motion.

As stated earlier, Ax represents the normal component of acceleration. Using Ax data, the angular velocity experienced by the accelerometer can be modeled using Equation 2.

\[ \omega = \sqrt{\frac{A_n}{R}} \] (2)
In Equation 2, $\omega$ represents angular velocity in radians per second, $A_n$ represents the normal component of acceleration in $m/s^2$, and $R$ represents radius in meters. For the horizontal trial, $A_x$ values were used as $A_n$ values, and the distance between the center of the wheel and the sensing element of the accelerometer, which was about 8 inches (0.2032 meters), was used as the $R$ value.

The angular velocity of every existing point on the wheel is the same, regardless of radial location, because all points on the wheel move the same number of degrees per unit time. Because of this, Equation 2 shows that moving the accelerometer closer to the center of the wheel (which would decrease $R$) must result in a proportional decrease in the normal component of acceleration (decrease in $A_n$) in order for angular velocity to remain constant at different radial locations.

When Equation 2 is solved using accelerometer data, the angular velocity of the entire wheel is being measured, not just the angular velocity of the accelerometer. Figure 3 shows a plot of the angular velocity of the wheel as a function of time.

![Figure 3: Plot of Angular Velocity as a Function of Time.](image)

Figure 3 shows that the angular velocity of the wheel decreases with respect to time, until the wheel comes to a stop shortly after the 20 second mark. Figure 3 also shows that the angular velocity of the wheel oscillates, which could be caused by the wheel being tilted. The trend line on the graph provides a mathematical model for angular velocity with respect to time, and is displayed in Equation 3 where $t$ is time in seconds.

$$\omega = 6.5892 - 0.2758t$$

While angular velocity is constant regardless of radial location, velocity changes with radial location. By knowing the angular acceleration of the wheel, the velocity of any point on the wheel can be determined, even though the accelerometer only gathered acceleration data for one point on the wheel. Equation 4 shows a relationship between velocity, denoted as $v$, radius, and
angular velocity. This relationship can be used to model the velocity of the wheel’s outer diameter.

\[ v = R \omega \]  \hspace{1cm} (4)

The wheel had a 24 inch diameter, so in order to model the outer diameter’s velocity, \( R \) must be set to 12 inches (0.3048 meters). Because \( \omega \) remains constant regardless of radial location, the same data displayed in Figure 3 for the accelerometer’s angular acceleration can be used to help model \( v \) for the outer diameter of the wheel. Figure 4 shows a plot of the velocity of the wheel’s outer diameter as a function of time.

\[ v = 2.0084 - 0.0841t \]  \hspace{1cm} (5)

It is important to note that neither Equation 3 nor Equation 5 are perfect mathematical models because their trend lines are not perfect, which is shown by the \( R^2 \) values on Figures 3 and 4. By setting the \( \omega \) in Equation 3 and the \( v \) in Equation 5 to equal zero, these models can predict the amount of time that it should take the wheel to stop spinning. These models predict that the wheel should spin for 23.9 seconds before coming to a stop. However, during the actual experiment, the wheel spun for about 20.7 seconds before stopping, which proves that these models are not completely accurate. Although these mathematical models are not perfectly accurate, they are good enough to be used for the rotational motion analysis of this lab.
Equation 3, which models angular velocity, can be used to create a model of angular acceleration, $\alpha$. This can be done by using the relationship between angular velocity, angular acceleration, and time that is shown in Equation 6.

$$\alpha = \frac{d\omega}{dt} = -0.2758 \text{ rad/s}^2$$  \hspace{1cm} (6)

By taking the derivative of Equation 3 with respect to time, it can be found that the angular acceleration of the wheel, $\alpha$, is $-0.2758 \text{ rad/s}^2$.

A model for the angular acceleration of the wheel could also be found by using Equation 5 and the relationships between acceleration (denoted $A$), velocity, time, radius, and angular acceleration shown in Equations 7 and 8.

$$A = \frac{dv}{dt} = -0.0841 \text{ m/s}^2$$  \hspace{1cm} (7)

$$\alpha = \frac{A}{R} = \frac{-0.0841}{0.3048} = -0.2758 \text{ rad/s}^2$$  \hspace{1cm} (8)

In Equation 7, the acceleration of the outer diameter is found by taking the derivative of Equation 5 with respect to time, and this acceleration is plugged into Equation 8 along with the 12 inch radius $R$ value of the outer diameter in order to find an angular acceleration of $-0.2758 \text{ rad/s}^2$, the same angular acceleration that was found using Equation 6. Both of these methods for finding an $\alpha$ value are equally valid.

The mathematical models found for angular acceleration and for angular velocity can be used to create a mathematical model for angular displacement, $\theta$, using Equation 9.

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$  \hspace{1cm} (9)

In Equation 9, $\omega_0$ represents initial angular velocity, which is found by plugging $t=0$ into Equation 3. In Equation 9, the value $-0.2758 \text{ rad/s}^2$, which was found in Equations 6 and 8, is used for angular acceleration $\alpha$. Equation 10 shows the mathematical model for angular displacement derived from Equations 9, 3, and either 6 or 8.

$$\theta = 6.5892t - 0.1379t^2$$  \hspace{1cm} (10)

By plugging in the time that the wheel spun (20.7 s according the accelerometer data) as a $t$ value, Equation 10 can be used to find the total angular displacement experienced by a point on the wheel. Plugging in $t=20.7$ gives an angular displacement of 77.3416 radians. This value of angular displacement can be used to determine how far the outer diameter of the wheel spun, which is also the distance that the wheel would have traveled if it had been rolling along the ground instead of spinning in the air. Equation 11 shows a relationship between radius, angular displacement, and linear displacement (denoted $S$) that can be used to find the distance that the wheel would have traveled had it been rolling.
\[ S = R\theta = (0.3048m)(77.3416\, \text{rad}) = 23.5737m \] (11)

So according to Equation 11, the wheel would have traveled 23.5737m had it been rolling on the ground.

By making the assumption that the radial position of the wheel’s center of mass is 11 inches from the wheel’s center axel, and by assuming that the accelerometer and the accelerometer’s clip are a point mass of 0.1 kg, the free body diagram shown in Figure 5 was constructed. The free body diagram shows the locations of the wheel’s mass and accelerometer’s mass, the wheel’s axel, and the force that acted on the wheel in order to initially spin it \( F_{\text{push}} \).

![Figure 5: Free Body Diagram of Force Acting on Wheel](image)

The moment of inertia about the wheel’s axel can be found by using the wheel’s mass, the accelerometer’s mass, the distance between the axel and the accelerometer’s center of mass \( (D_1) \), and the distance between the axel and the wheel’s center of mass \( (D_2) \). The relationship used to find the moment of inertia about the wheel’s axel, \( I_{\text{Axel}} \), is shown in Equation 12, where \( m \) represents mass.

\[ I_{\text{Axel}} = m_{\text{accelerometer}}D_1^2 + m_{\text{wheel}}D_2^2 = 0.0999 \, \text{kg} \cdot \text{m}^2 \] (12)

Solving Equation 12 gives a value of 0.0999 \( \text{kg} \cdot \text{m}^2 \) as the moment of inertia about the wheel’s axel. By using this moment of inertia, and by assuming that force was exerted on the wheel for a period of one second in order to initially spin it, the torque and force required to accelerate the wheel can be found using relationships between acceleration, angular acceleration, time, velocity, radial location, and moment of inertia.
The wheel accelerated from 0 m/s to 2.0084 m/s in a period of 1 second while it was being initially spun, so the acceleration of the wheel while it was being spun can be found using Equation 13.

\[
A = \frac{dv}{dt} = \frac{2.0084 \text{ m/s}}{1 \text{ s}} = 2.0084 \text{ m/s}^2
\]  

(13)

This acceleration of 2.0084 m/s² can be converted to angular acceleration by reusing Equation 8, as shown in Equation 14.

\[
\alpha = \frac{A}{R} = \frac{2.0084 \text{ m/s}^2}{0.3048 \text{ m}} = 6.5892 \text{ rad/s}^2
\]  

(14)

So, according to Equation 14, during the initial spinning process the wheel experienced an angular acceleration of 6.5892 rad/s². The torque applied to the wheel in order to spin it can be found using the relationship between this angular acceleration from Equation 14 and the moment of inertia found in Equation 12. This torque relationship is shown in Equation 15, where \( \tau \) represents the torque applied to the wheel in order to spin it.

\[
\tau = l\alpha = l_{axel} \alpha = (0.09999 \text{ kg} \cdot \text{m}^2)(6.5892 \text{ rad/s}^2) = 0.6583 \text{ N} \cdot \text{m}
\]  

(15)

So, according to Equation 15, a torque of 0.6583 N \cdot m was applied to the wheel for a period of one second in order to initially spin it. The force required to produce this torque can be found by using a relationship between torque, force (denoted as F), and the distance between the location the force is applied to and the center axel (denoted as D). This relationship is shown in Equation 16.

\[
F = \frac{\tau}{D} = \frac{\tau}{D_3} = \frac{0.6583 \text{ N} \cdot \text{m}}{0.3048 \text{ m}} = 2.1598 \text{ N}
\]  

(16)

Equation 16 shows that it must have taken 2.1598 N of force to initially spin the wheel, assuming that the force was applied for a period of one second.

If the wheel were to be spun by exerting a force on the spokes 1 inch from the wheel’s center axel instead of from its outer diameter, the torque required to spin the wheel would remain the same because neither the moment of inertia about the axel nor the angular acceleration required to bring the wheel up to the initial angular velocity would change. However, the force required to create this torque would increase. As shown in Equation 16, the applied force is dependent on the ratio of the torque to the distance between the location of the force and the wheel’s axel. Since torque is constant, if the location of the force decreased from 12 inches from the wheel’s axel down to 1 inch from the wheel’s axel, the force required to produce this torque to spin the wheel must increase by a factor of 12, up to 25.9173 N.

If the accelerometer were to be moved closer the center of the tire, \( D_1 \) from the Figure 5 free body diagram would decrease, which would cause the moment of inertia to decrease according to Equation 12. If the same torque were to be applied to the wheel in this scenario, the angular acceleration of the wheel would increase according to Equation 15, because the only way for \( \tau \) to remain constant when \( I \) is decreased is to increase \( \alpha \).
If \( \alpha \) were to increase, the maximum value of \( \omega \) would increase as well. This is because the wheel is being accelerated by a force for a period of one second up to its maximum angular velocity. Increasing the angular acceleration of the wheel during the initial one second long spinning process would increase the angular velocity of the wheel caused by the spinning process, which increases the maximum angular velocity of the wheel.

Increasing the maximum angular velocity of the wheel would likely increase the amount of time that it takes for the wheel to come to a stop after the initial spinning process. The plot of angular velocity as a function of time and its associated trend line, which are shown back in Figure 3, show that the wheel’s decrease in angular velocity as time passes can be modeled as linear. According to the plot and the trend line shown in Figure 3, the amount of time that it takes for the wheel to come to a stop should increase if the maximum angular velocity increases.

Therefore, if the accelerometer were moved closer to the wheel’s center, moment of inertia would decrease, causing the maximum angular velocity to increase, which would cause the wheel to spin longer.

For the vertical trial, the wheel was spun so that its diameter was perpendicular to the ground. Figure 6 shows a plot of the acceleration measured in each of the accelerometers three axes as functions of time when the wheel was spun vertically.

![Figure 6: Acceleration in each of the Accelerometers Axes as Functions of Time.](image)

For the vertical trial, the accelerometer’s Z-axis was always parallel to the ground, assuming that the wheel was not tilted. This is why Figure 6 shows very little acceleration measured in the Z-axis, apart from some small oscillations. Normal acceleration caused by the wheel’s rotational motion was measured in the X-axis, and tangential acceleration was measured in the Y-axis.

Unlike the data obtained for the horizontal trial, in which gravity was split between the three axis the same way regardless of the angular position of the accelerometer, the data obtained from the vertical trial has gravity constantly shifting between the accelerometer’s X-axis and Y-axis.
When the accelerometer is positioned at the highest point and the lowest point of its rotation (12 o’clock and 6 o’clock positions), the Y-axis is parallel to the ground, and all acceleration due to gravity is measured in the accelerometer’s X-axis. Similarly, when the accelerometer is positioned halfway between its highest and lowest points (3 o’clock and 9 o’clock positions), the X-axis is parallel to the ground, and all acceleration due to gravity is measured in the accelerometer’s Y-axis.

It is difficult to remove gravity from this data because the rate of the oscillations slows down as the angular velocity of the wheel decreases. In order to mathematically remove gravity from the data, a differential equation would have to be developed to describe how the acceleration due to gravity in the X-axis and Y-axis is periodically increasing and decreasing at a decreasing rate as the wheel slows down.

In real world applications involving rotating accelerometers, the accelerometer is often linked to a gyroscope that keeps track of the accelerometer’s orientation relative to gravity. Since the exact orientation of the accelerometer is kept track of by the gyroscope, simple vector calculus can be used to accurately subtract gravity from the accelerometer’s data.