ASEE's 123rd Annual • Conference & Exposition • New Orleans, LA • June 26-29, 2016

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Paper ID #16914

Espoused Faculty Epistemologies for Engineering Mathematics: Towards Defining "Mathematical Maturity" for Engineering

Mr. Brian E Faulkner, University of Illinois, Urbana-Champaign

Brian Faulkner is a graduate student at the University of Illinois at Urbana Champaign. His interests include teaching of modeling, engineering mathematics, textbook design, and engineering epistemology.

Dr. Geoffrey L Herman, University of Illinois, Urbana-Champaign

Dr. Geoffrey L. Herman is a visiting assistant professor with the Illinois Foundry for Innovation in Engineering Education at the University of Illinois at Urbana-Champaign and a research assistant professor with the Department of Curriculum & Instruction. He earned his Ph.D. in Electrical and Computer Engineering from the University of Illinois at Urbana-Champaign as a Mavis Future Faculty Fellow and conducted postdoctoral research with Ruth Streveler in the School of Engineering Education at Purdue University. His research interests include creating systems for sustainable improvement in engineering education, promoting intrinsic motivation in the classroom, conceptual change and development in engineering students, and change in faculty beliefs about teaching and learning. He serves as the webmaster for the ASEE Educational Research and Methods Division.

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1. Introduction

What role *should* mathematics play in an engineering student's education? A typical engineering undergraduate takes a five-semester course sequence of Calculus I, Calculus II, Calculus III, Linear Algebra and Differential equations (henceforth known as the *calculus sequence*). This sequence forms a rigid prerequisite structure for many engineering curricula. A single failing grade in one of these prerequisite courses can prevent a student from being able to progress into their engineering curriculum. Students may have to substantially delay graduation or leave engineering altogether before they have taken even one engineering course. Students with fewer high school educational opportunities, such as students of color, disabled students, or low socioeconomic status students, in particular, are thwarted by the calculus sequence [1]. Many are doomed before they even begin, since the timing of engineering courses assumes that all students are entering college "calculus ready".

Given the barriers that the calculus sequence poses to engineering retention, we must critically examine the rationale of faculty for requiring the calculus sequence. Why do engineering faculty require these courses? What do engineering faculty hope that their students will gain from the calculus sequence? During the authors' informal conversations to identify interventions to improve engineering mathematics instruction, faculty expressed a surprising opinion: They cared less about the techniques and concepts of calculus itself and more about the "mathematical maturity" that is gained by completing the sequence. In the following paper, we present our first steps to explore how engineering faculty define "mathematical maturity" and their rationalizations for requiring the calculus sequence. We present preliminary results from interviews with engineering faculty and discuss possible implications from these early findings.

2. Background

To guide our investigations into what faculty might define as mathematical maturity, we looked at prior research on expert-like mathematical behaviors. We found two particular constructs to be salient: mathematical epistemologies and symbol sense.

Epistemology is one's beliefs about knowledge and knowing (e.g., is knowledge immutable or is it constantly in flux?). Epistemology is contextual; students activate different epistemological beliefs depending on the discipline (mathematics or physics) or the context (seeking understanding or preparing for exams). Importantly, epistemology transcends the techniques or content knowledge of any one discipline. Students tend to develop more expert-like beliefs about knowledge over the course of college education. Since we expect the epistemologies of first- and second-year college students to be similar to that of graduating high school students, we list some epistemological stances concerning mathematics found to be common among high school students [2,3].

- Innate Ability: The belief that mathematical ability is fixed and unchanging and only geniuses can truly understand or create mathematics.
- Quick Learning: The belief that learning is a quick process, and furthermore, so is problem solving.
- Orderly Process: The belief that mathematical knowledge does not involve uncertainty or failure.
- Simple Knowledge: The belief that mathematical knowledge is disconnected, and that information gained in one lesson has no bearing on future or past lessons.
- Certain Knowledge: The belief that mathematical knowledge is not flawed or incomplete, especially material that is presented in class.
- Omniscient Authority: The belief that mathematical authority (usually a textbook or the instructor) forms the basis for mathematical truth.
- Practical Irrelevance: The belief that that the mathematics learned in school has no importance outside of the math classroom.
- Solitary Mathematics: The belief that mathematics is done by individuals alone, mathematics does not need to be communicated to peers.

In addition to epistemology, “symbol sense”[4] may capture part of what faculty mean by “mathematical maturity.” Symbol sense describes a student’s understanding of the power of symbols and notation. Students with symbol sense can engineer symbolic relationships without explicit calculation (such as the exponential form of the decaying current in an RC circuit). These students continuously check for symbol meaning as they work through problems. Students with symbol sense can choose an appropriate symbolic representation, and recognize the different roles symbols can play in different contexts (x and x’ might represent a displacement and its time derivative in freshman mechanics, but a source location and a target location in third year electromagnetics). As students move from high school to college, the “symbol load”[5] on them increases drastically with many new symbols (such as greek letters and outer product symbols), new types of objects (such as vectors and operators), and disciplinary disagreement on what symbol to use. For example, students in freshman mechanics are reluctant to solve problems symbolically (rather than plug in numbers), despite the encouragement of their instructors [6].

Despite the relative ubiquity of the calculus sequence in engineering curricula, there has been little research justifying this particular choice of content and order of presentation for engineering students. Little research has investigated whether engineering faculty actually expect students to know calculus content, or are instead hoping students adopt certain behaviors by passing through the trial of calculus. While some have explored how calculus topics are used further in differential equations[7] and how they are applied in engineering[8], these studies have focused primarily on the mathematical content. Less attention has been paid to the habits of mind and attitudes towards mathematics that engineering faculty wish their students to develop. While we know that mature epistemologies and symbol sense are expert-like behaviors, we don't know if faculty are implicitly referring to these habits of mind when they discuss "mathematical maturity." Furthermore, we don't know if the calculus sequence is engendering mature epistemologies or symbol sense in engineering students. There may be a fundamental mismatch between what mathematics instructors want students to leave their courses with and what engineering instructors expect students to enter their courses with are well documented [9].

Our preliminary discussions with faculty indicated that mathematical maturity, not calculus content, was the true prerequisite for their courses. Since the engineering faculty perception of mathematical maturity is not well-defined, our **first research question** is "How do engineering faculty define "mathematical maturity?" To investigate what mathematics knowledge is truly essential, our **second research question** is "What mathematics topics are considered essential knowledge by engineering faculty?"

3. Methods

To develop a rich understanding of how engineering faculty define mathematical maturity and what mathematical knowledge they desire of their students, we took a qualitative approach that relies on focused interviews. We interviewed faculty who taught any course in the College of Engineering requiring a course from the calculus sequence as a direct prerequisite or corequisite. We chose this criterion because these courses are expected to be the most directly impacted by the calculus sequence. We narrowed the sample pool by selecting only those faculty members who taught these courses during the Fall 2014 or Spring 2015 terms (a pool of 60 faculty). This ensured that these faculty members remembered their experiences teaching the selected courses. On occasion participants encouraged us to interview another faculty member who failed to meet our criteria but were considered to be important voices in our campus's dialogue about this topic. Two such faculty were added to the sampling pool. By the end of the project, we plan to interview at least two faculty members from each of the 12 engineering departments at our institution. Such a sample can provide a voice for each of the disciplines and allow for some comparison within a department.

We conducted hour-long, semi-structured interviews with engineering faculty. The interview protocol contained an initial set of questions and possible follow-up questions, but the conversations were allowed to develop naturally. Ideas from symbol sense, epistemology, and "mathematical maturity" were explicitly probed for near the end of the interview if the faculty member did not mention them in their responses to other questions. Faculty were compensated for their time by entry into a raffle for a $200 gift card.

Some basic interview questions included:

- What are the prerequisite mathematics courses for your engineering course?
- What are the prerequisite math knowledge and skills?
- What is missing from your students' mathematical abilities entering your course? Which abilities are satisfactory?
- Do you consider attitudes towards mathematics important for your course?
- Are your students comfortable using mathematical notation?
- What does the phrase 'mathematical maturity' mean to you?

As interviews are still ongoing, the results presented below are initial the initial impressions and observations of the interviewer from interviews with twelve faculty. We will use formal qualitative analysis methods once interviews are completed.

The sampling of faculty to interview excluded many late stage courses. As a result, the following observations may downplay the importance of advanced mathematics in third and fourth year courses. Additionally, because the sample has focused on only one institution so far, our results may not be reflective of faculty perceptions at other institutions. Specifically, many of our engineering department faculty are applied mathematicians and scientists who switched to engineering rather than engineers by training. Additionally, our institution is a top five, highly selective engineering program which may skew the expectations of our faculty on students' abilities.

4. Results

Faculty described a range of desired mathematical skills and attitudes when describing mathematically mature students. A mathematically mature engineering student has fast, effortless algebra skills, and can apply notation flexibly. This student can set up problems from physical descriptions to solve on a computer. This student approaches mathematics with curiosity, unafraid to try techniques they aren't certain are perfectly correct. A mathematically mature engineering student uses math as the language for communicating engineering ideas. This student always searches for applications of any mathematics he or she is shown, and uses these applications to understand mathematics more richly. In the following section, we elaborate on each of these themes.

4.1. Fluent Algebra skills

"The main reason for having the Calc I prerequisite is so that students have a reasonable fluency with precalculus. Basically the entire course uses precalculus."-Computer Science faculty member

Many faculty who teach freshman/sophomore courses don't stress the actual *calculus* content of calculus. They want algebraic fluency. They want algebraic manipulations to be fast, effortless, and intuitive for students. These faculty just want their students to have solid algebra skills, and hope that those skills have been acquired by the end of calculus. These faculty emphasize that despite their course listing calculus as a prerequisite, that "strong algebra is what they want." When these instructors expected certain calculus content, it was often only the simplest cases such as the differentiation of small polynomials. Students need a "decent command of just more basic skills of being able to do some algebra, to solve an equation, doing some simple calculus to take the integral."

4.2. Effortless manipulations

"Normally the students know all the rules. The question is how fast can they apply them."-Computer Science faculty member

Faculty are concerned with their student's algebraic skills, even at a highly ranked engineering program. This complaint fades away for instructors of third-year courses. Perhaps the intervening courses instill algebraic fluency in students or have eliminated students that did not already have these skills. Specifically, faculty are concerned with the *speed* at which students can perform algebraic manipulations. Faculty teaching later courses also want simple calculus calculations to be fast and second nature, students "should be able to differentiate a polynomial without thinking about it." These faculty stress that this speed and ease with algebraic (or simple calculus) manipulations so that students "can allocate [their] mental resources to understanding the hard part of the problem."

4.3. Translating real world problems into equations

"[A mathematically mature student is] able to set up a differential equations from conservation, able to translate the real world problem to the differential equation."-Bioengineering faculty member.

Mathematical proficiency for engineering students is less about the techniques of solving problems, and more about the ability to set up those problems. The ability to apply solution techniques is a necessary skill, but is less important than setting up equations from a verbal description. Particularly, mathematically mature students can use universal principles like conservation laws, common in all engineering disciplines, to develop a basic equation to describe how a system works. Faculty emphasize that the difficult, meaningful part of their use of mathematics is setting up equations. Solving the equations is of secondary concern.

4.4. Ability to use computational tools to solve mathematical problems

"Students should be able to work the simple problem on paper so they get some intuition. Most of the problems they're going to deal with are bigger than anything they could do on paper, so they need to understand how that translates to computer and visualization tools."- Bioengineering faculty member

Engineering faculty today have little interest in their students' ability to solve tricky integrals and differential equations through analytic manipulation. They view these techniques as no longer necessary in the age of computers. Engineering faculty are far more interested in their students being able to set up and model these situations computationally since "frankly, they're going to stick it in Mathematica or Wolfram Alpha". Using mathematics computationally is much more important to engineers than tricky analytic techniques. "Do they need to know how to solve differential equations? NO! We just need programming. Programming is the problem." The time spent on trigonometric integrals and the method of integrating factor appears to be in vain. Engineering faculty want their students to be able to solve *simple* differential equations and integrals, to get intuition for examining more complex systems. Only the most basic, simplest cases, such as differentiation of exponential functions, integration of polynomials, or solving the first order y'=ky differential equation, are truly of import.

4.5. Fluency with notation

"We have to have someone come in and sweep across and remove the fear of notation, seeing problems in different ways."-Bioengineering faculty member

"One thing that goes into math maturity is knowledge of basic notational issues." Faculty report their students as being deeply uncomfortable with notation and symbols, perhaps a lack of **Symbol Sense**. The students are "very tied to notation", and cannot recognize when two systems of notation from different disciplines represent identical ideas. Particularly, engineering faculty claim that "they don't explain these things" in mathematics courses, such as when to use dots or primes for derivatives, or the difference between variables and parameters in a model. "They may have derivative of ln(x) in calculus, but when they get the derivative of a bunch of symbolic constants in front of ln(r), it looks like a different problem to them." Students aren't capable of flexibly applying notation and recognizing that one idea can be written in many different ways in different contexts.

4.6. Willingness to explore multiple solution approaches

"[Students have a] certain skittishness in actually trying different tools, in starting some random direction to start a problem."-Physics faculty member

The engineering faculty notice that their students are uncomfortable with 'taking a shot in the dark' with mathematics. Few will attempt a solution without knowing ahead of time they are doing it correctly. In the words of one faculty member, "Math comes across as very slick. I encounter that that shapes an expectation that everything that comes down the pike is completely polished and rigorous that covers all the corner cases." Students are reluctant to just try something, and if it doesn't work out, change tack. This behavior indicates the epistemological stance of **Orderly Process**. A more mathematically sophisticated student, by contrast, is "quite happy blundering around in the forest" looking for a solution. This expectation may be shaped by the time-limited nature of exams, in which there is no room for uncertainty or exploration.

4.7. Perceive mathematics as the language of technical communication

"I would hope that coming out of my course that they appreciate how differential equations are the language of physiology."-Bioengineering faculty member

Faculty stress the nature of mathematics as a language of technical communication. Schoenfeld[2] mentions that the belief that mathematics is something done by individuals alone undermines belief in mathematics as a form of communication. Faculty say that one of the "telltale signs [of mathematical maturity] is language in describing math". Furthermore, they lament the lack of mathematical communication skill in their students. Some students "don't want to use words, they sometimes use funny patterns of dots and shorthand, sometimes they don't even do that. For some of them it's just this bare sequence of equations." This may be a manifestation of a **Solitary Mathematics** epistemological stance held by students.

4.8. Able to interpret the real world implications of mathematical expressions

"It is really a matter of them making a link between reality and the math." -Agricultural Engineering faculty member

Engineering faculty are deeply concerned about their students' ability to apply mathematics. To engineering faculty, good math knowledge is *connected* to context and to application. Students "don't recognize that what we're talking about in my class is the same mathematical operation as what they... learned to do in a methods of integration unit." This can be interpreted as the epistemology of **Practical Irrelevance** held by the students. Students don't believe that the mathematics they have learned in math classes is connected "to chemistry and biology and measurement." For some faculty, this is an explicit course goal, "The target is getting them to feel the connection between physiology and math." One faculty member, when asked if they could guarantee their students had one skill or habit entering their course, said "What I'd like, is that they're used to and in the habit of asking what they're learning is for." Students don't believe that what they learn in mathematics courses is connected to their engineering careers, and faculty view this as a great concern.

4.9. Confidence when using mathematics

"[Fear of math] seems beaten into students. not enough time is spent on justifying why it is interesting to look at, more is spent on the testable nuggets." -Computer Science faculty member

Engineering faculty say that mathematically mature students are "able to understand concepts like infinity and a limit and not be scared." Faculty at all levels and in many departments speak of students being skittish around mathematics. Both theorists and practitioners want students to be less scared, and more curious. They want their students to "appreciate the power of mathematical modeling." And this fear has dire consequences, in one professor's words "They don't know why and they get a C [in Calculus] and they barely get through and they hate math as a result. They're very intimidated by it and it becomes part of their self image about their limitations. There's this thing called math and I've got to stay away from that." Engineering faculty perceive their students comfort and confidence with mathematics to be key issues. Furthermore, faculty claim students have this "attitude of fear of math because it was disconnected" from applications.

5. Conclusions

Faculty conceptions of mathematically mature students align with parts of the previous research on mature epistemologies and symbol sense. Faculty described some of the novice epistemologies such as practical irrelevance, solitary mathematics, and orderly process as epistemologies that they wanted students to proceed past. Similarly, faculty alluded to symbol sense when they discussed fluency with notation and the ability to perform effortless manipulations of equations. Future studies will need to explore more about why faculty found these aspects of epistemology and symbol sense to be so salient.

Reflecting the conversations that sparked this investigation, faculty were not particularly concerned about whether students could learn specific calculus techniques or concepts. Rather, faculty were surprisingly deeply concerned about students' algebra skills. This result is striking as it provides evidence that the current structure of the calculus sequence may be misaligned with what training students actually need to be successful in the engineering curriculum.

Another unexpected finding was that faculty expressed a desire for greater computational training of students. This finding arose without any prompting in our interview protocol. The faculty argued that since the analysis of most “real problems” is intractable, students should be learning more about how to use computational tools to solve problems rather than on how to solve tricky integrals. This shift in emphasis from hand calculations to computer-based computations may create new challenges. Learning programming, and furthermore how to program mathematics, is known to be a challenging task.

Given these opinions of engineering faculty, it may be time to reexamine the mathematics curriculum. The engineering sciences curriculum, spurred on by the Grinter Report, has held sway over engineering mathematics instruction for decades. Engineering has changed, computation is now a dominant tool and it may have altered what skills are necessary for success: techniques such as integration have become automated and engineers must increasingly wrestle with analytically intractable systems. Future work and curriculum design must carefully consider the role of mathematics and computational tools in engineering curriculum and instruction.

6. Acknowledgements

This work was supported by the National Science Foundation under grant DUE 1544388. The opinions, findings, and conclusions do not necessarily reflect the views of the National Science Foundation or the author’s institution.

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