Abstract – This paper demonstrates the Least Mean Square (LMS) adaptive filter for adaptive noise (interference) removal. A measured signal contains an unknown signal of interest and an interference signal. The goal is to design an LMS based adaptive noise canceller that adaptively removes the maternal heartbeat signal from the fetal electrocardiogram signal. This Least Mean Square based adaptive filter will remove the interference signal from the measured signal by using a reference signal. The reference signal used is highly correlated with the interference signal. The filtration is achieved by designing a Least Mean Square adaptive filter with a specific order and step size that will ensure the adaptation of the filter to converge after few seconds of adaptation.

Index Terms—Least Mean Square, MME, MFE

I. INTRODUCTION

The Least Mean Square Algorithm will use two measured signals to perform the adaptive filtering.

Measured Maternal Electrocardiogram (MME): The Maternal Electrocardiogram signal is a recorded signal obtained from the chest of the mother. The heart rate for this signal is approximately 89 beats per minute, and the peak voltage of the signal is around 3.5 millivolts [8].

Measured Fetal Electrocardiogram (MFE): The measured fetal electrocardiogram signal is a recorded signal obtained from the abdomen of the mother. This signal is dominated by the maternal heartbeat signal that propagates from the chest cavity to the abdomen [9]. The heart of a fetus beats noticeably faster than that of the mother. The heart beat rate range is from 120 to 160 beats per minute for fetus [8]. The amplitude of the fetal electrocardiogram is also much weaker than that of the maternal electrocardiogram [8]. The peak voltage of the Measured Fetal Electrocardiogram signal is around 0.25 millivolts.

Both the recorded Measured Maternal Electrocardiogram (MME) and Measured Fetal Electrocardiogram (MFE) are stored as a data file and contain the values of three variables:

MFE: the measured fetal electrocardiogram

MME: the measured maternal electrocardiogram

F: the sampling frequency of the ECG data.

Before proceeding further a general block diagram of signal filtering process is shown in figure 2 where it can be observed that signal during its transmission from source to destination is interrupted by undesired signal as indicated as noise in figure 1.

Received Signal represents the original source and noise. This signal is then passed through filter algorithm such as Finite Impulse Response or any other Adaptive Algorithm depends upon the nature of Noise.

II. ADAPTIVE FILTER

Adaptive filter algorithm works in two ways as described by [1] and figure 3 shows a basic and typical block diagram of Adaptive filter. Where reference signal is taken as the Measured Maternal Electrocardiogram MME and observed that the heart beat signal pass through the mother’s body as the signal pass through a Least Mean Square (LMS) filter. While the d(n) is taken as the Measured Fetal Electrocardiogram (MFE) signal and is the mixture signal with the mother’s heart beat in the womb including the baby’s heart beat signal with noise. After the Measured Fetal Electrocardiogram (MFE)
minus the filtered Measured Maternal Electrocardiogram (MME) by Least Mean Square (LMS) filter is the baby’s heart beat with some noise.

Figure 3. Adaptive Filter

III. LEAST MEAN SQUARE ALGORITHM

A simple to implement and easy convergence is the LMS algorithm where convergence is dependent upon the step size. Figure 4 shows the block diagram of Least Mean Square Algorithm.

Figure 4. LMS algorithm

\[ y(n) = \sum_{k=0}^{M-1} w_k u(n-k) \]

\[ w \] is the weight also known as filter coefficients, \( k \) shows the order of filter.

In our design we used Finite Impulse Response (FIR) filter and made it adaptive in nature. FIR filter is always more stable than IIR Filter [2].

The LMS filter mimics the mother’s body from the chest to the stomach. The Adaptive LMS filter used has 8 as the order of the filter and \( w \) coefficient is initialized.

IV. DESIGN AND IMPLEMENTATION

In our LMS Adaptive Filter the algorithm follows below 5 steps and is also shown in figure 3 as a block diagram

1. Set the order of the filter to 8 and initialize the \( w \) coefficient.
2. Compute the predicative output \( y(n) \) which is the filtered output signal.
   \[ y(3) = \sum_{k=0}^{2} w_k u(3-k) = w_2 u(3) + w_1 u(2) + w_0 u(1) \]
3. Calculate the estimation error
   \[ e(3) = d(3) - y(3) \]
4. Compute the new \( w \) coefficients as the adaptive new weights.
   \[ w(n+1) = w(n) + \mu u(n)e(n) \]
5. These weights by circulate computing new filtered output signal until the total input MME signal is filtered.

Figure 5 is the step wise algorithm implantation snapshot with Filter Order of 3 for demonstration purpose.

The filtered Measured Maternal Electrocardiogram MME signal is the heart beat signal of mother transmitted to the stomach. Measured Fetal Electrocardiogram MFE cancels the filtered MME. The cancellation error is the baby’s heartbeat. The error signal is about 0.26 mV level which is the baby’s heart beat signal added with little noise.

Figure 5. LMS Block Diagram First Iteration

\[
\begin{align*}
\begin{bmatrix}
    w_2(2) \\
    w_1(2) \\
    w_0(2)
\end{bmatrix}
&= \\
\begin{bmatrix}
    w_0(1) \\
    w_1(1) \\
    w_2(1)
\end{bmatrix}
&+ \mu e(3)
\begin{bmatrix}
    u(3) \\
    u(2) \\
    u(1)
\end{bmatrix}
\end{align*}
\]

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>VARIABLE USED AND THEIR DEFINITIONS</th>
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<tr>
<td>Symbol</td>
<td>Definitions</td>
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<tr>
<td>( \lambda )</td>
<td>Maximum of Eigen values</td>
</tr>
<tr>
<td>( d(n) )</td>
<td>Measured Fetal Electrocardiogram</td>
</tr>
<tr>
<td>( u(n) )</td>
<td>Measured Maternal Electrocardiogram</td>
</tr>
<tr>
<td>( w )</td>
<td>Filter Coefficients</td>
</tr>
<tr>
<td>( k )</td>
<td>Filter Order</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Condition for Convergence</td>
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<td>( e(n) )</td>
<td>Error Signal</td>
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<tr>
<td>( y(n) )</td>
<td>Filtered Output</td>
</tr>
</tbody>
</table>

| \begin{align*}
    w_2(1) & = 0 \\
    w_1(1) & = 0 \\
    w_0(1) & = 0 \\
    u(1) & = y(3) \\
    u(2) & = y(4) \\
    u(3) & = y(5) \\
    u(4) & = y(6) \\
    u(5) & = y(7) \\
    u(6) & = y(8) \\
    u(7) & = y(9) \\
    u(8) & = y(10)
\end{align*} |
The above equation will update the filter weights and new weights \( w_k(2) \) where \( k \) is the filter order. With new filter weights the algorithm moves on to the next iteration as shown by figure 6.

\[
\begin{align*}
\text{Figure 6. LMS Block Diagram Second Iteration} \\
y(4) \text{ and } e(4) \text{ are calculated with the following formulas.} \\
y(4) &= \sum_{k=0}^{2} w_k u(4-k) = w_0 u(4) + w_1 u(3) + w_2 u(2) \\
e(4) &= d(4) - y(4) \\
\end{align*}
\]

The present weights \( w(2), e(4) \) and \( u(2), u(3), u(4) \) will yield further new weights as shown by the following formula

\[
\begin{align*}
\begin{bmatrix}
  w_0(3) \\
  w_1(3) \\
  w_2(3)
\end{bmatrix} &= 
\begin{bmatrix}
  w_0(2) \\
  w_1(2) \\
  w_2(2)
\end{bmatrix} + \mu e(4) \\
&= 
\begin{bmatrix}
  u(4) \\
  u(3) \\
  u(2)
\end{bmatrix}
\end{align*}
\]

Where \( \mu \) is the step size and has to satisfy the condition of \( 0 < \mu < 1 / \lambda \) where \( \lambda \) is the eigen value of autocorrelation matrix.

The proposed Least Mean Square adaptive filter algorithm is implemented in Matlab using the finite impulse response filter algorithm. This algorithm was also tested in TMS320 DSK6713 kit as shown in figure 7 with algorithm written in C-Language.

\[
\begin{align*}
\text{V. RESULTS AND DISCUSSION} \\
\text{We will show the results in form of graphs starting with Figure 8 that shows the mother’s heartbeat as the filter output with the LMS filter order 8.} \\
\text{Figure 8. Mother, Fetal and Noise (Order 8)} \\
\end{align*}
\]

Fetal heartbeat is shown as an estimation error in figure 8 and noisy signal.

Figure 9 shows the magnitude of the fetal heartbeat as \( Y=0.269 \) in the close view of estimation error graph.

\[
\begin{align*}
\text{Figure 9. Fetal Heartbeat Close View} \\
\text{This estimated value of fetal heartbeat of 0.269 is very close to the actual value of the fetal heartbeat which is 0.25 as observed by figure 9.} \\
\text{Figure 10 shows the coefficients of the LMS filter known as}
\end{align*}
\]
weights of the filter and denoted by N having order 8.
The mother’s heartbeat, fetal heartbeat and the noisy signal in
figure 8 is calculated by the filter having order N=8 as shown
by figure 10.
It is noted that there is a relationship between the order of the
filter and the effect of the output signal.
Outputs of mother heartbeat and fetal heartbeat was re-
calculated with order N=10 as can be seen by Figure 11.

Figure 10. Weights of the Filter with Order 8

Figure 11 shows the mother’s heartbeat as the filter output,
fetal heartbeat as estimation error and the noisy signal
calculated with the filter order of 10.

Figure 11. Mother, Fetal and Noise (Order 10)
The close view of the fetal heartbeat in figure 12 shows that
the magnitude of the fetal heartbeat is Y=1.258 which is much
weaker as compared to the magnitude achieved at the order of
8 where the magnitude value is Y=0.269.

Figure 12. Fetal Heartbeat Close View

Weights of the LMS adaptive filter can be seen in figure 13
calculated with order 10.

Figure 13. Weights of the filter with Order 10

VI. CONCLUSION

It is noted that there is a relationship between the order of
the filter and the effect of the output signal. Changing the filter
order will result in less noise. Increasing the order of the filter
will give less magnitude of the fetal heartbeat. During the
order changing process, it is found that the optimum range of
filter order is 6 to 8 and the time that the filter begins to work
normally is all about 500 unit times.
Noise amplitude is quite higher than the fetal heartbeat
amplitude, which is 0.25 mV. The weights are not shown
collectively rather it is shown at different order. The method
of calculating the ensemble weights and errors is similar.

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